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PHYSICS OF TECHNOLOGY

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ANALYTICAL BALANCE

Measurement, Errors, and Mechanical Equilibrium.

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ANALYTICAL BALANCE

A Module on Measurement, Errors, and Mechanical Equilibrium

FVCC

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ANALYTICAL BALANCE

PREFACE TO THE STUDENT

Section A of this module will teach you some of the physics concepts and principles of an analytical balance. You will also learn how to use and read various kinds of balances and scales. The approach will be nonmathematical, and certain questions will be raised by your laboratory work first; then you will be led to forming some answers to these questions. In Section A you will not always get complete answers to all of the questions raised in your lab work. The answers you get will describe what happens, but not when, or how much. To get *quantitative* answers, we must use numbers, make measurements, and write some equations.

Section B of this module will provide some quantitative answers about physics concepts and principles of the analytical balance. These answers will be formed by what is called an *empirical* approach. In this approach, you make measurements of one quantity for different values of another, with all other related quantities held constant. Then the resulting relationships are said to be empirical.

To discuss *why* certain answers are or are not correct, we must develop models (theories) for what we observe. Then we must compare observations with the theory, and we should try to extend the theory to predict effects which have not yet been observed. Finally these new predictions should be tested with experiments.

Section C of this module will examine physics theories of the analytical balance, as well as applications of concepts and principles, both empirical and theoretical.

This module has been designed so that you may complete your study of it at the end of Section A, Section B, or Section C. If you already have the skills, knowledge, and understanding taught in Section A, you may begin the module with Section B. The module post-test is divided into three parts, A, B, and C, corresponding to the three module sections.

GOALS FOR SECTION A

The following goals state what you should be able to do after you have completed this section of the module. These goals should be studied carefully, as you proceed through the module, and as you prepare for the post-test. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item *like* the one given, you will know that you have met that goal. These goals and test items permit you to test yourself and thereby know when you have learned what is expected of you from this section of the module.

These goals represent the minimum of what is expected of you. We hope that your success in meeting these goals will encourage you to achieve many others not stated here. Answers to the items appear immediately following these goals.

1. *Goal:* Demonstrate an understanding of the concept of mass.

   Item: Two carts are tied together by a string. Between the carts is a compressed spring. When the string is cut, the carts move away from each other. The following table shows the speeds of the two carts, after cutting the string, in the cases of two different springs.

Spring	Speed of Cart 1	Speed of Cart 2
1	40 cm/s	100 cm/s
2	10 cm/s	25 cm/s

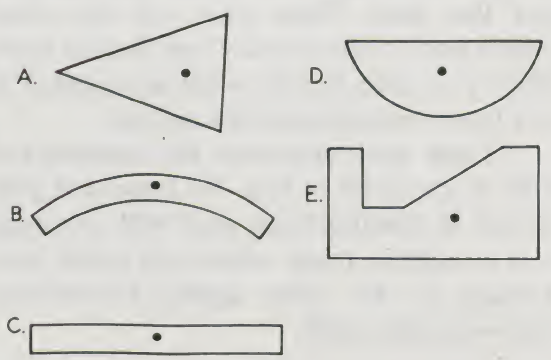
   Which cart has the greater mass, and how much greater is its mass than that of the other cart?

2. *Goal:* Understand the relationship between weight and mass.

   Item: The mass of one object is 6 mass units. At a certain place, its weight is 24 weight units. Another object has a mass of 4 units. What is its weight?

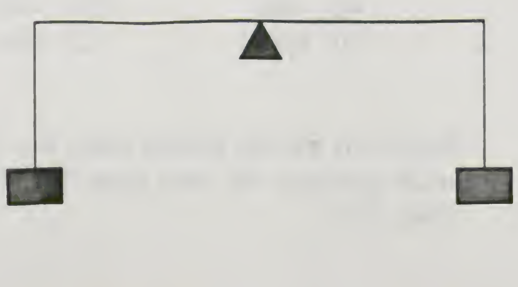
3. *Goal:* Demonstrate an understanding of the concept center of gravity (or center of mass).

   Item: The figure shows flat pieces of tin which have been cut to the shapes shown. For which piece of tin is the center of gravity not correctly located?



4. *Goal:* Demonstrate an understanding of the principles of a first-class lever.

   Item: Suppose that you have a first-class lever made of a very light-weight material (much less weight than that of the masses supported at the ends of the lever). Initially, the lever is in horizontal equilibrium as shown in the sketch. If the mass on the right is increased while that on the left is decreased, which way must the fulcrum be moved to keep the system in horizontal equilibrium?



5. *Goal:* Know the meaning of the terms *stable* and *unstable equilibrium,* and be able to tell when a lever is in one of these states.

   Item: The following are descriptions of four different lever systems:

   a. Most of the mass of the lever is located below the fulcrum.
   b. Most of the mass of lever B is located above the fulcrum.
   c. If lever C were given a slight angular displacement, it would continue to rotate in the same direction.
   d. If lever D were given a slight angular displacement, it would oscillate about its original position.

   Which systems are in stable equilibrium, and which are in unstable equilibrium?

6. *Goal:* Demonstrate an understanding of the principle of the lever used as a balance.

   Item: A first-class lever is used as a balance. The system is in horizontal equilibrium when a certain amount of mass in on each pan. When a small mass is added to one pan, the lever comes to rest at a new equilibrium position. Suppose this procedure is followed for three different balances. The following information is given about the balances or the deflection due to a given small mass.

   a. The masses of the balance beam systems are all equal.
   b. The center of mass of the beam system of Balance I is at a greater distance below the fulcrum than the center of mass of Balance II is below its fulcrum.
   c. Balance I has a resulting pointer deflection of five units.
   d. Balance III has a resulting pointer deflection of two units.

e. All balance pointer lengths are equal.
f. The pan arm lengths of all three balances are equal.

Where is the center of mass of Balance III relative to that of Balances I and II? Which balance is most sensitive? Which is least sensitive?

7. *Goal:* Demonstrate an understanding of the principle of buoyancy.

Item: An iron object weighs the same as an aluminum object when weighed in air. How do the weight readings of these two objects on a spring scale compare when "weighed while they are immersed in water"?

Answers to the Items Accompanying the Preceding Goals

1. Cart 1; Mass of Cart 1 = 2.5 times mass of Cart 2

2. 16 weight units

3. B

4. To the right

5. Stable: A, D. Unstable: B, C

6. The center of mass of Balance III is at a greater distance below the fulcrum than that of either Balance I or Balance II; Balance II is most sensitive; Balance III is least sensitive.

7. The weight reading of the aluminum object is less than that for the iron object because the buoyant force on the larger aluminum object is greater than that on the iron object.

SECTION A

A Qualitative Approach to the Physics Concepts and Principles of the Analytical Balance

HISTORY OF THE BALANCE

The balance is a device we use to measure the weight or mass of an object. The distinction between weight and mass will be discussed later in this module, but for now the two terms will be used interchangeably. To weigh something means to compare its weight or "heaviness" to the weight of something else. A simple way to compare the heaviness of two objects is to hold one in each hand. An unbalance is sensed as a feeling that one object is heavier than the other, unless the difference between their weights is too small to be detected by this method. For greater sensitivity and accuracy, many different types of weighing instruments have been invented throughout history. The earliest recorded use of a mechanical balance was in Egypt about 5000 B.C. Figure 1 shows the features of one of these early balances.

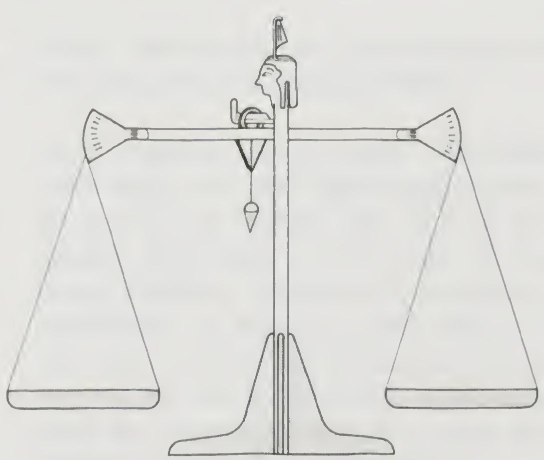


Figure 1.

The balance consists of two pans hung from the ends of a beam. The beam is suspended or pivoted at the center, forming two equal distances to the suspension points of the weighing pans. When the beam stays level, the weight in one pan is said to equal the weight in the other. This type of balance is called an *equal arm balance* and is widely used today.

KINDS OF BALANCES

Many other types of mechanical balances have been invented in the 7000 years since then. Not all of these later balances were just refinements of the early Egyptian one; many of them differed basically in the way balance is achieved. Among the other different types of mechanical balances or scales are the *spring balance,* the *torsion balance,* the *magnetic balance,* and the *unequal arm balance.* Some common types of balances are shown in Figure 2. Examples of the spring balance include the fish scale and the common bathroom scales.

PRINCIPLES OF OPERATION OF DIFFERENT BALANCES

The principles of operation of a spring balance are familiar to most people; the amount of stretch or compression of a spring by a weight depends on the heaviness of the weight. The torsion balance determines the weight of an object from the amount of twist of a tiny quartz fiber. This type of balance can be extremely sensitive and is used to weigh very small objects. In this module we will study equal arm and unequal arm balances, whose principles of operation are the same.

HISTORY OF WEIGHT UNITS

How is the weight of an object determined on a mechanical balance? As mentioned earlier, when the beam of an equal arm balance remains level, the weight on one side equals the weight on the other. One side of

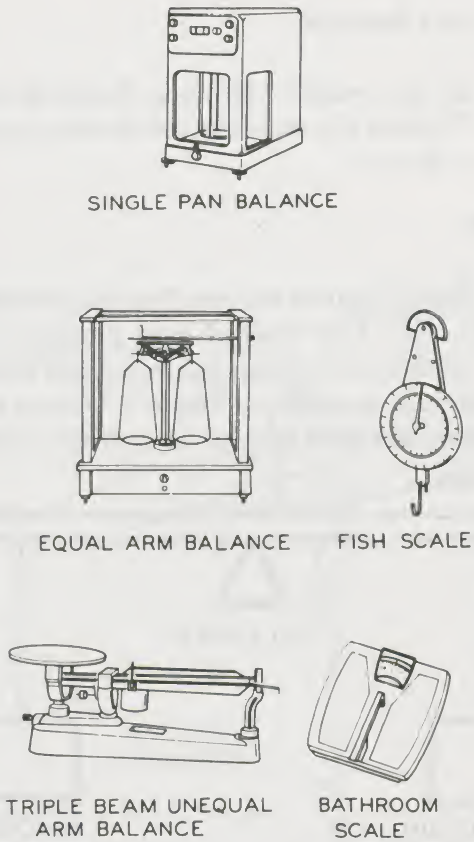


Figure 2.

the balance must contain a known weight. This known weight is referred to as a "standard of weight," and is arrived at by general agreement. Known weights are measured in chunks of equal size (and fractions of chunks) called standard *units.* In ancient times each tribe had its own standards. Since the early balances were used for trading goods, like grain crops, it is not surprising that the oldest recorded standards were based on handfuls of grain. Particular standards were spread by means of trading among tribes and nations, and through invasions during wars. After the fifth century B.C., the Greek *drachma* or "handful of coins" was a common standard of weight. The Romans introduced a unit of weight, the "pondo libra" or *pound by weight,* which was later to spread to Europe and England. A variation of this unit is one of the present standards of weight, the pound. The abbreviation lb, for pound, comes from the Roman libra, which meant balance or scale. There have been many variations of the pound, several of which are still in use today. The *troy pound,* used for weighing precious metals such as gold and silver, is smaller (5760 grains) than the familiar *avoirdupois pound* (7000 grains).

METRIC UNITS

The late 18th century saw the introduction in Europe of the metric system of weights and measures. The metric system was based on decimals, with a standard unit of length called the *meter* (m). The meter was defined as one ten millionth of the distance along a certain arc on the earth's surface. The metric system remained almost unchanged until the first International System of Units and Standards was adopted in its present form in 1960. These S.I. *(Système International)* units have now been officially adopted by most countries of the world and by every major nation except the United States. The standard of mass in the S.I. system is called the *kilogram.* A kilogram has a weight of about 2.216 lb. *

*While weight should properly be measured in units called *newtons* (discussed later in this module), we will follow common laboratory practice and frequently refer to the weight as being so many grams.

EXPERIMENT A-1. Weights and Balances

This is an exploratory experiment. Use the appropriate work sheets from the back of the module. Write answers to questions, and complete the tables and graphs.

Part A

Four balances or scales have been provided for this experiment. Each has been labeled with a letter, A, B, C, or D.

1. Classify each balance or scale according to the type mentioned in Figure 2 on page 5.

2. Figure out how to use and read each balance or scale; then find the weight of each of two pennies on each balance or scale.

3. For each balance or scale, what is the difference in weight readings for the two pennies?

4. Are the weight differences found in Step 3 the same for all balances and scales used? How do you account for these results?

Some balances are designed to weigh heavy objects and some to weigh light objects. The heaviest load which can be weighed on a balance is called the *capacity* of the balance.

5. From an examination of each balance, determine its capacity. Record those capacities.

6. You have been given two weights. Describe how you can determine if the weight of one of these objects exceeds the capacities of one of the balances.

7. For the balances which have capacities larger than the weight of either of these objects, find the weight of each object and their difference.

8. Do the weight differences found in Step 7 follow the same pattern as those found in Step 4?

Part B

Many balances are based on the principle of the *lever.* One kind of lever consists of a rigid straight rod supported at a point called the *fulcrum* as shown in Figure 3. Objects can be suspended from the rod on each side of the fulcrum.

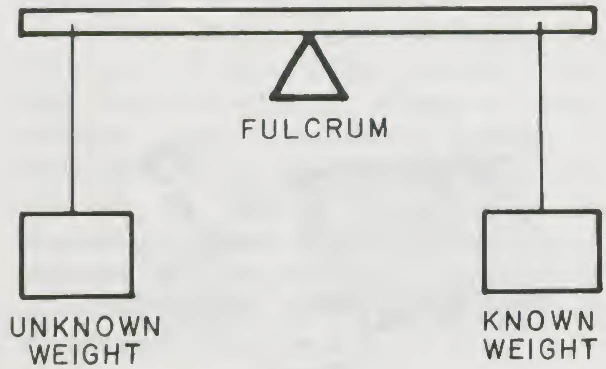


Figure 3.

Let us use a meter stick as a lever, with adjustable weight supports and an adjustable beam weight. This apparatus is shown in Figure 4.

1. Remove the weight holders and adjustable supports, B, from each end of the lever. Move the meter stick in the fulcrum apparatus to the point where the meter stick stays horizontal when supported at the fulcrum. In this position, the lever is balanced about the fulcrum. How far is the fulcrum from the left end of the lever?

2. Place the weight-holder supports on each side of the lever at equal distances from the fulcrum. Attach the weight holders to the supports. Support this lever system at the fulcrum and adjust the fulcrum position along the meter stick until the system will balance horizon-

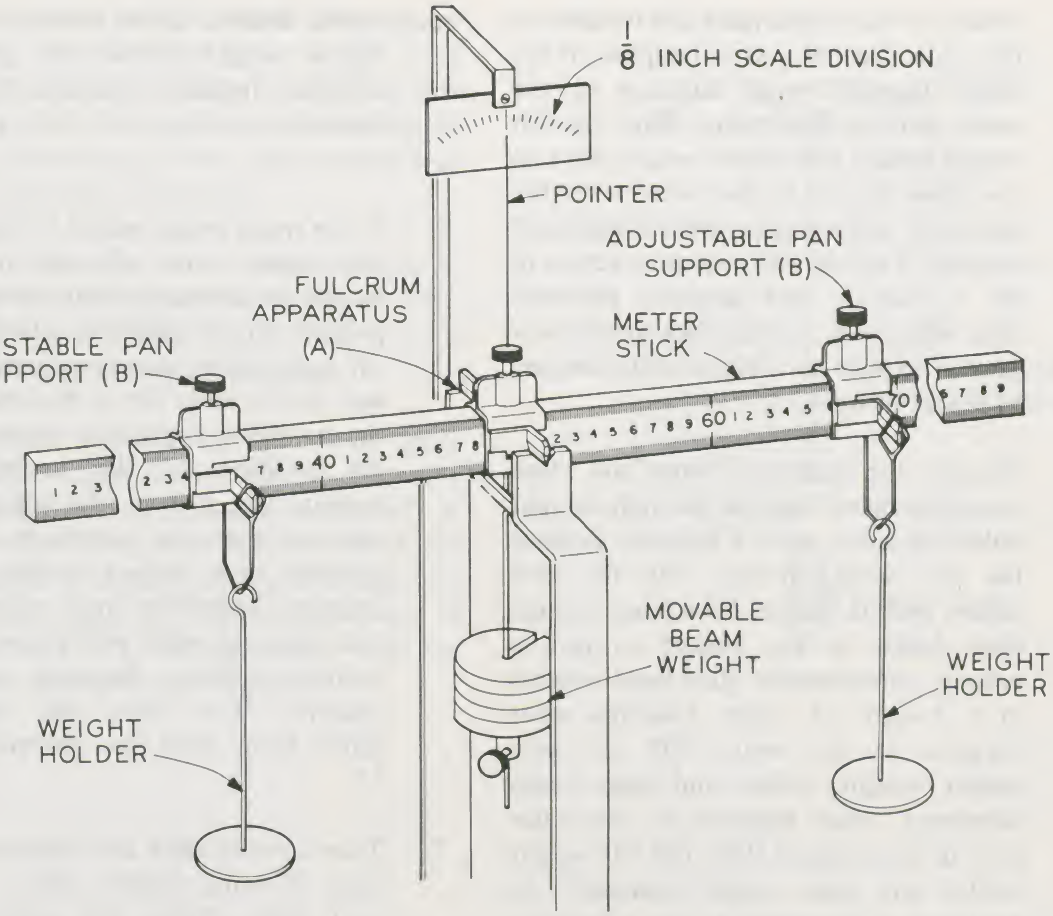


Figure 4.

tally. Holding the lever horizontal, place some metal weights on the right-hand weight holder. Describe what happens to the lever when it is released.

3. How much weight do you have to place on the left weight holder to get a balance?

4. Remove the weights from each weight holder and move the fulcrum support 10 - 15 centimeters (cm) toward the right-hand weight-holder support. Set the lever on the fulcrum. Describe what happens to the lever system. Add the weights needed to achieve balance.

5. Remove the weights from the weight holders and repeat Step 4 for two or three other positions of the fulcrum along the meter stick.

In your own words, describe how this lever system is affected by the kinds of changes you have made in the fulcrum.

Part C

Let us now investigate how the weight of an object appears to change when it is immersed in a fluid.

1. Put water into a large beaker until it is about three-fourths filled. With the lever system in balance, and with 200 - 250 grams on each weight holder, bring the beaker of water up under the left weight

holder, totally immersing the weights on that side. Describe what happens to the lever. Describe what happens to the water level in the beaker. With the left weight holder still under water, what do you have to do to the weights on the right side of the lever system to again get balance? Perform the necessary action to get a balance, and describe precisely what was done. In your own words, how do you explain the effects of immersing an object in water?

2. Remove the beaker of water and place enough weights back on the right weight holder to again achieve balance. Replace the left weight holder with the wire basket weight holder. Now place enough glass beads in the basket to give a balance. Immerse the glass bead weights in a beaker of water. Describe what happens to the lever. With the wire basket weight holder and glass beads immersed, what happens to the water level in the beaker? With the left weight holder and glass beads immersed, do whatever is necessary to get a balance, and describe precisely what you did. How do these results compare with what you observed in Step 1? How do you explain the difference?

Part D

The lever system you have been given has an adjustable weight located below the fulcrum. By adjusting this weight, you can raise or lower some of the mass of the beam system.

1. Place the fulcrum apparatus so that the lever system is in horizontal balance when the weight-holder supports are about equally distant from the fulcrum. Adjust the position of the movable beam weights attached to the fulcrum apparatus so that this weight is as low as possible below the point of support. Place about 150 grams (g) on each weight holder, but keep the system in horizontal balance. Now place a small additional weight (about 10 g) on one weight holder. Describe what you observe.

2. If the small weight added in Step 1 made the pointer move off scale, replace this weight by a weight small enough for the pointer to be deflected but not move off scale. Next remove the added weight and set it aside for a moment. Return the system to horizontal balance. Adjust the position of the movable beam weights attached to the fulcrum apparatus so that this weight is as high as possible with respect to the point of support. Take the small weight which had been set aside and place it on one weight holder. Describe what you observe. How does this observation differ from what you observed in Step 1?

3. Take a meter stick and balance it on the edge of some support, like a pencil or foot ruler. When the meter stick is balanced, place a small weight on one end of the meter stick. Describe what you observe.

4. Why do you suppose the behavior of the system in Step 3 differs as it does from the behavior of the system in Steps 1 and 2?

Part E

Let us now examine another phenomenon which you will later see is related to the principles of the balance.

1. Cut a piece of heavy cardboard into any irregular shape, perhaps like that shown in Figure 5. Support the cardboard with a straight pin stuck into a point near the edge.

   From the pin suspend a thread with a paper clip tied to its end. Hold the pin in

a horizontal position. Mark the position of the thread on the bottom edge of the cardboard. Draw a line connecting this mark with the point of support. Repeat this procedure for two other points near the rim of the cardboard.

Mark the point where these lines cross. Now try to support the piece of cardboard at this point on the tip of your finger. Describe what happens.

2. Is there a point in the meter stick balance which corresponds to the point you have marked on the cardboard? If so, where is it? What do you think the connection is between the identification of this point for the irregular piece of cardboard and the experiment with the meter stick and lever apparatus?

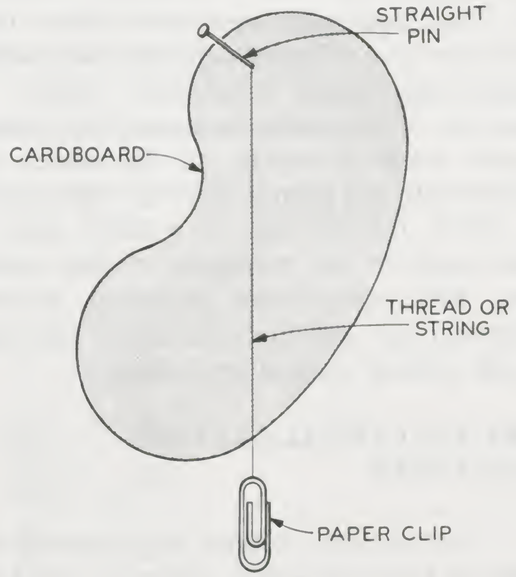


Figure 5.

WHAT IS AN ANALYTICAL BALANCE?

In Experiment A-1, you found that balances have different capacities and sensitivities. The term "analytical balance" is reserved in scientific language for balances which weigh accurately to the nearest ten thousandth of a gram (0.0001 g). This amount is about 1/5 the mass of a single grain of table salt. In the remainder of this module we will discuss the analytical balance. However, the principles discussed will apply to all equal or unequal arm balances.

THE ANALYTICAL BALANCE AS A LEVER

An analytical balance is essentially one kind of lever. We define a *lever* as a rigid rod pivoted at a point called the *fulcrum.* When the fulcrum is located *between* the points at which the two forces are acting, the system is called a *first class lever.* The essential features of a balance are illustrated in Figures 6 and 7.

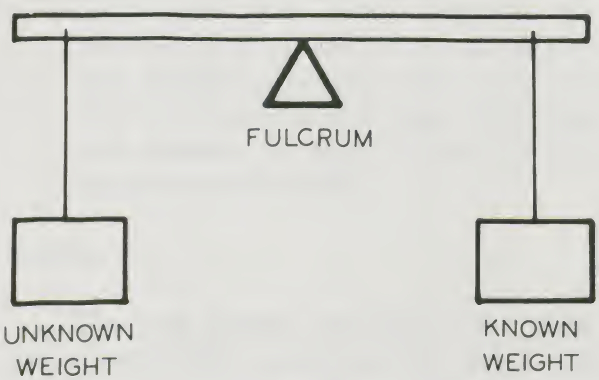


Figure 6.

CONCEPT OF EQUILIBRIUM

The principle of operation of the lever has been known for many centuries. In fact, the famous Greek scientist Archimedes of Syracuse (287-212 B.C.) stated the law of the lever in his writings. Experience had taught him that important factors in achieving balance were not only the size of the weights, but also their distances from the fulcrum. When the system is balanced, and the beam has no tendency to tilt one way or the other, it is said to be in *equilibrium.* If the two weights are equal, and their distances from the fulcrum are also equal, then the lever is in equilibrium, since there is no greater reason for the system to rotate clockwise than to rotate counterclockwise.

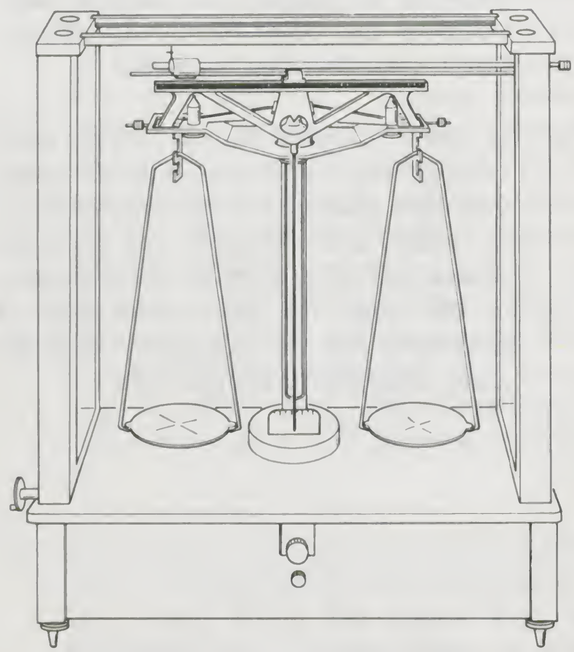


Figure 7.

PRINCIPLE OF THE SUBSTITUTION BALANCE

Another type of analytical balance is becoming widely used in present-day laboratories. This type is called the single pan or substitution balance. The principle of this balance can be understood from Figures 8A, B, and C. Suppose a see-saw has a child sitting on one side, with enough weights on the side so that the see-saw stays horizontal.

Then suppose another child, who is smaller than the first, gets on the side of the see-saw with the weights. His side goes down to the ground, and there is an unbalance.

Now suppose that some of the weights are taken off until the see-saw is again balanced. The total weight *taken off* would be the same as the weight of the smaller child.

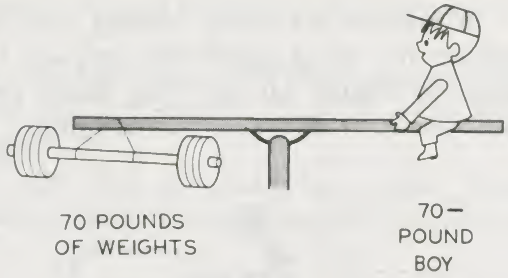


Figure 8A.

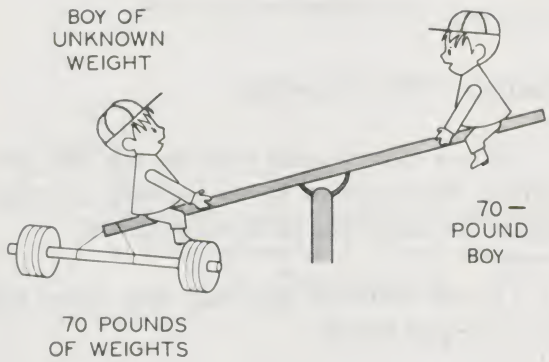


Figure 8B.

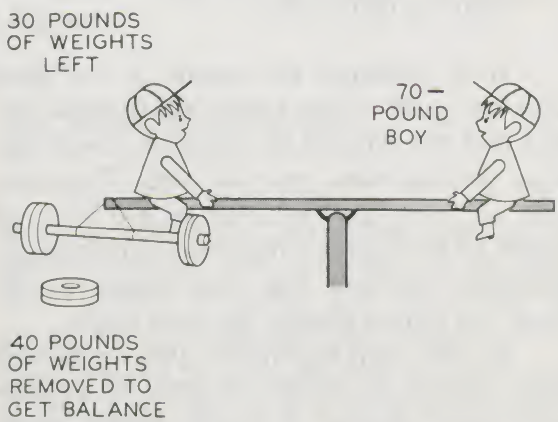


Figure 8C.

As with the see-saw, the single pan balance has a fixed weight on one arm, which the user generally can't get to. On the other arm are removable weights and a pan for the object to be weighed. The balance is in equilibrium when the pan is empty. The sample to be weighed is placed on the pan, so that the system is not in balance. Weights are then removed from the arm supporting the sample until equilibrium is again attained. Then the total weight removed is equal to the weight of the sample. Usually the weights are removed by turning knobs on the instrument, and the total is automatically registered on a dial which is read by the user.

Figure 9 shows a typical single pan substitution analytical balance.

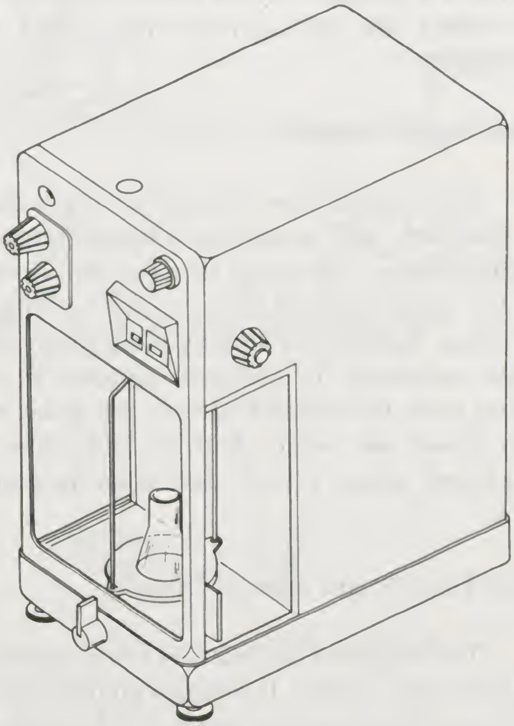


Figure 9.

The single pan balance has become popular because of the ease and speed with which it can be used. Ordinarily, a trained operator can make a weighing on the single pan balance much more rapidly than with a double pan balance. Before we study the balance further, we will examine the concepts *weight* and *mass*.

WHAT IS WEIGHT?

The *weight* of an object is the name given to the pull of gravity exerted on the object. Thus weight is a force. Suppose we take a spring balance and pull on it. We stretch the spring, a small stretch for a weak pull, and a larger stretch for a stronger pull.

We call such an action (one that stretches a spring, bends a fishing rod, makes a chair seat sag) a *force*. When we hang an object on the spring balance and the balance is stretched downward, we say that the force of gravity is pulling on the object, resulting in the stretching of the spring. Thus the stretch of the spring is a measure of the force of gravity on the object. We call this force the "weight" of the object.

How Weight Changes

If we could move around on the surface of the earth with some object hanging from a spring balance, we would find that the stretch of the spring changes very slightly from place to place. It decreases slightly if we go up on a high mountain. It increases slightly as we travel from the equator toward the poles. On the moon, we would find the stretch very much less, about 1/6 of what is on the earth.

What Does Weight Depend Upon?

The weight of an object is not a property of that object alone. It involves an interaction with another body (for example, the earth or moon). We call this interaction "gravitational," and it is an observed fact that the strength of this interaction (the "weight" of an object) varies from place to place.

ANOTHER KIND OF INTERACTION

Let us consider an entirely different experiment; one that we can perform which does not depend on the force of gravity. Suppose that we tie two carts together with a compressed spring between them as shown in Figure 10.

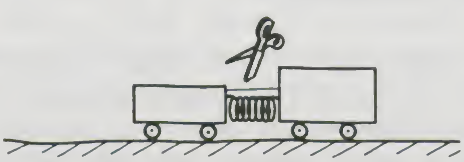


Figure 10.

If we cut the string holding the carts together, the spring pushes the carts apart, as shown in Figure 11, and they move away from each other on the horizontal surface.

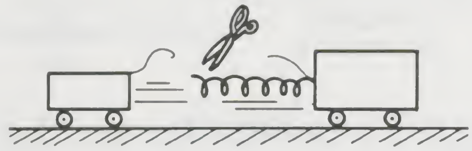


Figure 11.

Results of This Interaction

When experiments such as this are performed, for example, by two people on roller skates, the following facts are observed:

1. If the carts are identical, they move off at equal speeds.

2. If the carts are not identical, the heavier one always moves off at a lower speed than the lighter one.

If we measure the speeds of the carts after the push of the spring has finished, we find that the ratio of the speeds is always the same for the same pair of carts. This ratio does not vary with location on the earth. It would be the same on the moon. We could, in principle, perform this experiment in far space and obtain exactly the same result.

In your own experience, you know that some objects are harder to move or to stop than other objects. A heavily loaded truck can't speed up as rapidly as a small car, even with a much larger engine in the truck. You also know that it is much harder to stop such a heavy truck than a smaller car. If you have ever tried to move a heavy load on the mover's dolly, you know how hard it is to get the load moving. And once moving, you have experienced the problem of stopping it, or changing its direction of motion. This property of objects has been called *inertia*, and it is the same property we are now considering in these interactions.

Another Way of Looking at the Interaction

Another way of obtaining this same ratio is to set up two stops, A and B, as shown in Figure 12.

By trial and error, one could find a starting point C such that after the spring is released the carts strike obstacles A and B simultaneously. The distances traveled by the carts are then Distance 1 for cart 1 and Distance 2 for cart 2, and the ratio of these distances is exactly the same as the ratio of the speeds we mentioned before.

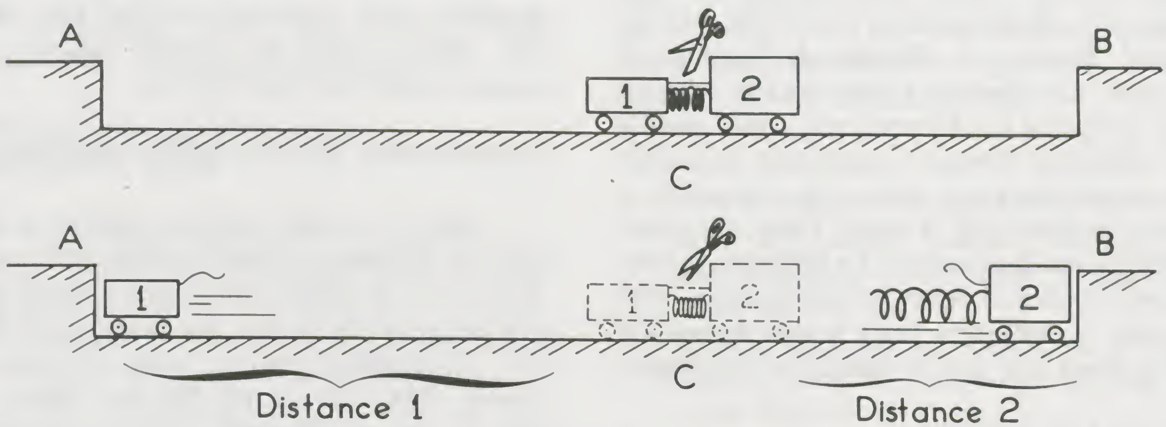


Figure 12.

Question 1. Explain in your own words why the ratio of distances is the same as the ratio of speeds.

THE MEANING OF MASS

The ratio of Distance 1 to Distance 2 is a property only of the interacting bodies (if we ignore friction). When Distance 1 is larger than Distance 2, we say that the "mass" of the second cart is *greater* than the mass of the first cart, and we call the ratio of Distance 1 to Distance 2 the ratio of mass 2 to mass 1. This can be written,

$$\frac{\text{Distance 1}}{\text{Distance 2}} = \frac{\text{mass 2}}{\text{mass 1}}$$

Thus, if we say that the mass of the first cart has a value of one unit, and adopt it as our "standard," we are always comparing the mass of the second cart with that of our standard mass, which has a value of one. We call the ratio of Distance 1 to Distance 2 the "mass of the second cart." For,

$$\frac{\text{Distance 1}}{\text{Distance 2}} = \frac{\text{mass 2}}{\text{mass 1}}$$

$$= \frac{\text{mass of cart 2}}{1}$$

$$= \text{mass of cart 2}$$

If we add bricks to the second cart (keeping the first cart the same) and now find that the ratio of Distance 1 to Distance 2 is twice what it was before, we say we have doubled the mass of the second cart. If, instead, we remove material from the second cart and find that the ratio of Distance 1 to Distance 2 is half of what it was initially, we say we have cut the mass of the second cart in half. If the ratio of Distance 1 to Distance 2 is 1, we say the mass of the second cart has a value of one; it has the same mass as our standard.

Problem 1. Suppose two carts are pushed apart by a spring as described. If cart 1 travels 25 cm while cart 2 travels 100

cm, which cart has the larger mass? How much larger is its mass? If cart 2 has a mass of one unit, what is the mass of cart 1?

Standard Units of Mass

The internationally accepted unit of mass is the *kilogram*, which is defined as the mass of a platinum block kept at the International Bureau of Weights & Measures in France. All reference masses used on balances are compared to this standard, either directly or indirectly through intermediate standards. In much laboratory work, it is convenient to use a smaller unit of mass. Thus, one *gram*, which is one-thousandth of a kilogram, is commonly employed. An even smaller unit is often useful: A *milligram* (mg) is one thousandth of a gram and one millionth of a kilogram.

MASS AND WEIGHT

We have seen that mass and weight are two entirely different concepts. "Mass" is measured by the motion imparted to free objects interacting directly with each other (as through a spring) and has nothing to do with the pull of gravity. "Weight" is our name for the force of gravity on the object in question. But, if we double the mass of a body, it turns out that its weight is also always exactly doubled.

Comparing Weights and Masses

When we balance "known" and "unknown" objects against each other on an equal arm balance, we are comparing the pull of gravity on each side. Thus we are comparing the weights of the two objects instead of measuring the weight directly, as we would be doing on a spring balance. Since we double the weight if we double the mass (i.e., since weight and mass are proportional to each other), we are simultaneously comparing the masses of the two objects and comparing their weights, when we balance them against each other.

Thus the operation of "weighing" on an equal arm balance can be regarded either as a comparison of weights in the given location or as a comparison of masses, depending on which quantity we happen to be interested in.

When in a weightless situation, masses are determined using what is called an inertial balance. This balance compares the effort needed to speed up an unknown mass with the effort needed to speed up a known standard mass. Astronauts in the U.S. Sky Lab Project used an inertial balance to measure masses of foods they ate.

Proportionality between Weight and Mass

There is a nice example which is a lot like the difference between weight and mass. As you know, money and ounces of gold are not the same thing. But for many years the U.S. Government set the value of gold at exactly \$35 per ounce. We can write an equation relating the dollar value of a piece of gold to its weight in ounces,

$$\text{Dollar value} = 35 \times \text{weight in ounces}$$

We would say that the dollar value is not the same as the gold weight, but that the two quantities are *proportional.* The constant factor relating these quantities (35 in this example) is called a *proportionality constant.*

In the same way, weight and mass are proportional, but are not the same thing. On the earth these quantities have the relationship,

$$\text{Weight} = 9.8 \times \text{mass in kilograms}$$

where the weight is a force measured in units called *newtons* (N). When the pull or weight is expressed in the more familiar units of pounds, this equation becomes:

$$\text{Weight in pounds} = 2.2 \times \text{mass in kilograms}$$

We say that weight is *proportional* to mass but not identical to it.

Question 2. How much will a kilogram weigh on the moon? How much will an object which

weighs 1 lb on the earth weigh on the moon?

Question 3. What is the mass of a standard kilogram on the moon? How would your own mass on the earth compare with your mass on the moon? How would your weight on earth compare with your weight on the moon?

From this point on we will use the symbol "kg" to represent kilogram.

CENTER OF GRAVITY AND CENTER OF MASS

In Part E, Experiment A-1, you located a point on an irregularly shaped piece of cardboard. If the piece of cardboard is balanced on a fulcrum at this point, the tendency of the weight of one part of the cardboard to turn the cardboard is countered by the tendency of the weight of another part of the cardboard on the other side of the point of support. All the turning tendencies of the weights of various parts of the cardboard are balanced out. We call this point about which turning tendencies are balanced the *center of gravity.* Because weight and mass are proportional, we can also call this point the *center of mass.*

Exercise. Using the procedure followed in Part E, Experiment A-1, find the location of the center of gravity of pieces of cardboard of the following shapes (cut these shapes from a large sheet): circular; square; rectangular; triangular. Where is the center of mass for each shape?

Question 4. Where would be the center of gravity of a doughnut shaped piece of cardboard? Where is its center of mass?

STABLE AND UNSTABLE EQUILIBRIUM

In Part D, Experiment A-1, you observed that a given added mass gave a certain deflection of the balance when the center of mass of the beam system was close to and directly below the fulcrum (when the movable weight was as high as possible). Then, when the center of mass of the beam system was further below the fulcrum (when the movable weight was as low as possible), the deflection was less. When you balanced a meter stick on the end of a ruler and then placed a small weight on the end of the meter stick, the meter stick rotated all the way around until it fell off the supporting ruler. From those demonstrations, and some others similar to it, we can conclude that

1. A lever system is "unstable" (turns over, does not *oscillate** around an equilibrium position) if the center of mass is *above* the fulcrum.
2. A lever system is in equilibrium at *any* angular position if the center of mass is at the fulcrum.
3. A lever system oscillates around an equilibrium position if the center of mass is below the fulcrum. This is called a "stable" condition.
4. For a given unbalancing weight in a lever system, the size of the angle for which equilibrium is achieved depends on the center of mass of the system. As the center of mass of the rigid part of the lever is lowered below the fulcrum, the equilibrium angle becomes smaller.

Question 5. A frequently seen parlor toy or conversation piece has the following form: An iron-rod figure of a man with weights in his hands stands on a support as shown in Figure 13. When the man is given a push, instead of tipping over, he oscillates back and forth for a long time. Explain the stability of this system in your own words. Describe what would happen to its behavior if the weights were raised or lowered.

Question 6. Would the man of the previous question be stable if his arms were in sockets at the shoulders, which permitted the arms to hang vertically regardless of how the body would be tipped?

*Look up this word in a dictionary if you do not know what it means.

SENSITIVITY OF A BALANCE

You have observed that a lever system is stable only when the center of gravity is below the fulcrum. You have also observed that a small added weight will deflect a balance a greater amount when the center of gravity of the balance beam system is closer to the fulcrum than when the center of gravity is farther below the fulcrum. We define balance *sensitivity* in terms of this deflection for a given added mass; the greater the deflection, the greater the sensitivity.

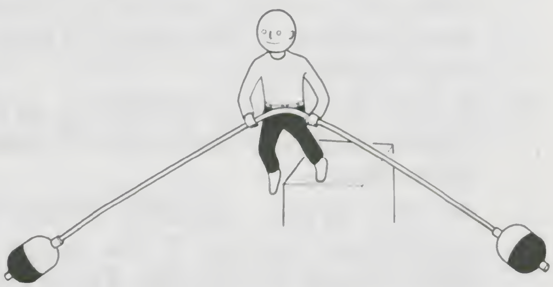


Figure 13.

BUOYANT FORCE

When the weights on one end of a balance are immersed in water, as you observed in Experiment 1, the balance tips as though the weight had decreased on the side which was immersed. We could achieve horizontal equilibrium of the balance again by removing weights from the other side. When we change the immersed weights from metal to glass, the amount by which the weight on that side appears to decrease is even greater. We must then remove more weights from the other side to achieve horizontal equilibrium again. We notice also in this experiment that the water level is raised somewhat when the metal weights are immersed. When the metal weights are replaced by glass of the same weight and then immersed, the water level is raised even more. From demonstrations such as these we conclude that:

1. An object which is immersed in water has an upward force exerted on it by water.
2. Since the object sinks in the water, the upward push due to the water is less than the weight of the object.
3. When the object is immersed in water, it pushes aside a certain amount of the water.
4. An object made of heavy (dense) material is pushed upward less by water when immersed than an object *of the same weight* which is made of a lighter material.
5. An object made of light (less dense) material having the same weight as an object made of heavy (more dense) material pushes aside more fluid when immersed than does the object made of heavy material.

The upward push exerted by the fluid on an object immersed in the fluid is called the *buoyant force.*

Question 7. Suppose you immerse an object in a fluid where the upward push is greater than the weight of the object. Describe what would happen when the object is released beneath the surface of the liquid. Name some materials which actually behave in this way.

Question 8. Suppose that we could show that air behaves as a fluid and exerts a buoyant force on objects placed in it. If you have an equal arm balance in horizontal equilibrium, then remove the metal weights on one arm and replace them by glass beads of equal weight, what would happen to the balance? (Assume that the glass beads were initially compared on a balance with the metal weights in a vacuum chamber, where no air was present.)

Question 9. A substitution-type analytical balance has removable weights made of stainless steel. If air can be considered a fluid, will the balance reading be larger or smaller than the true weight of the substance?

SUMMARY

The following statements summarize the concepts, definitions, and principles you have learned so far in this module.

A *standard* is a certain agreed upon measure of a physical quantity. (There are agreed upon standards of mass, length, time, temperature, etc.)

The name of a standard is referred to as a *unit* of the quantity (one kilogram, one meter, one second, etc.).

A *measurement* is made when we compare some quality with a standard.

Two related quantities are *proportional* when repeated measurements of the two quantities always have the same ratio. This ratio is called the *constant of proportionality.*

The *mass* of one object can be compared with the mass of another by comparing the changes in motion of the two objects when they interact with each other (for example, through a compressed spring).

A *kilogram* is the unit of mass in the International System of Units.

A *gram* is one thousandth of a kilogram. A *milligram* is one thousandth of a gram.

Force is an action, which, for example, will stretch a spring or bend a rod. (This statement is a fact, not a definition of force.)

The *weight* of an object is the force gravity exerts on it.

Weight is proportional to mass. As an equation, this proportionality can be expressed as:

Weight in newtons (on earth) = 9.8 $\times$ mass in kilograms

Weight in pounds (on earth) = 2.2 $\times$ mass in kilograms

The point in a flat object at which the object can be balanced is called the *center of gravity.* Because mass and weight are proportional, this same point can be called the *center of mass.*

A *lever* is a rigid rod pivoted at a point called the *fulcrum.*

When the fulcrum is between the points at which forces are acting on a rigid rod, the system is called a *first class lever.*

Horizontal equilibrium of a first class lever used as an equal arm balance is achieved when the weights on each end are equal.

Horizontal equilibrium of a first class lever in the form of an unequal arm balance is achieved when the weight on the shorter arm is greater than the weight on the longer arm.

A first class lever is in *unstable equilibrium* if, when given a slight push, the lever continues to rotate in the same direction (or falls off the support).

A first class lever is in unstable equilibrium when the center of mass of the rigid part of the lever system is above the fulcrum.

A first class lever is in *stable equilibrium* if the lever oscillates about its original position when given a slight angular displacement and then released.

A first class lever may be in stable equilibrium when the center of mass of the rigid part of the lever system is below the fulcrum.

The center of mass of the rigid part of a first class lever used as a balance is always designed to be below the fulcrum.

The *sensitivity* of a first class lever used as a balance is the amount by which the lever is deflected to a new equilibrium position by a given weight added to one side.

The sensitivity of a first class lever used as a balance is increased as the center of mass of the system is raised closer to the fulcrum.

Objects immersed in a fluid push aside some of the fluid.

The upward force exerted by a fluid on an object immersed in the fluid is called a *buoyant force.*

An object made of heavy (more dense) material is pushed upward less by a buoyant force than an object of equal weight made of lighter (less dense) material.

When weighing a substance which is made of heavier (more dense) material than that of the removable weights of a substitution-type balance, the balance readings will be too high due to buoyancy.

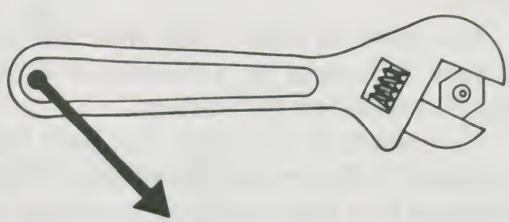
When weighing a substance which is made of lighter (less dense) material than that of the removable weights of a substitution-type balance, the readings will be too low due to buoyancy.

GOALS FOR SECTION B

The following goals state what you should be able to do after you have completed this section of the module. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item *like* the one given, you will know that you have met that goal. (Answers follow these goals.)

1. *Goal:* Know the meaning of the terms *line of action* and *lever arm.*

   Item: The illustration shows a wrench with a force being applied in a direction indicated by the arrow. Sketch in the *line of action* and the *lever arm* about an axis through the center of the nut for this force. Label this lever arm.



2. *Goal:* Understand and be able to use the definition of *torque.*

   Item: A first-class lever is in horizontal equilibrium. An object having a mass of 200 g is added to a weight holder which is 30 cm to the left of the fulcrum. The value of the acceleration of gravity, *g*, is 9.80 meters per second^2 (m/s^2). What is the value of the resulting torque, and what effect does the torque have on the lever system?

3. *Goal:* Understand and be able to use the additive property of torque.

   Item: A lever system has several weights suspended at different points on each side of the fulcrum. Suppose that the torques are 100 newton-meters, 30 newton-meters, and 50 newton-meters due to weights on the left of the fulcrum. On the right side the torques are 80 newton-meters, 60 newton-meters, 20 newton-meters, and 70 newton-meters.

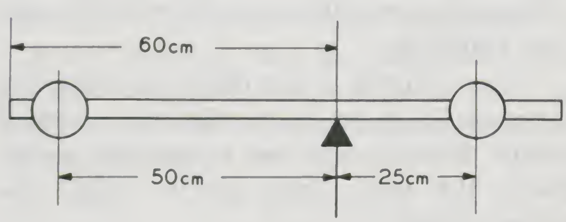
   a. What is the total torque tending to rotate the system clockwise?
   b. What total torque tends to rotate it counterclockwise?
   c. What is the net torque, and what effect will it have on the system?

4. *Goal:* Know how to use symmetry to locate center of mass.

   Item: a. At what point can we consider the total mass of a doughnut as being concentrated?
   b. A baseball?
   c. A meter stick?

5. *Goal:* Understand how to calculate and combine torques to compute net torque, using the concept of center of mass.

   Item: A metal rod having a length of 100 cm and a mass of 4 kg has spheres of the same metal attached as shown in the drawing. Each sphere has a mass of 5 kg, and they are located as shown. Use 9.80 meters per second^2 as the acceleration of gravity. Taking all of the masses into account, what is the torque about the fulcrum due to the weight of this system?



6. *Goal:* Know how to use experimental data to determine an empirical law.

Item: When the pressure of a fixed volume of a gas is measured for several different values of absolute temperature, the data graphs as a straight line passing through the origin. If the pressure, p, is on the vertical axis and temperature, T, is on the horizontal axis, and if the slope has a value of 890 (in appropriate units), what is the relationship of pressure to temperature, written as an equation?

7. *Goal:* Know how to solve problems involving empirical laws which are direct proportions.

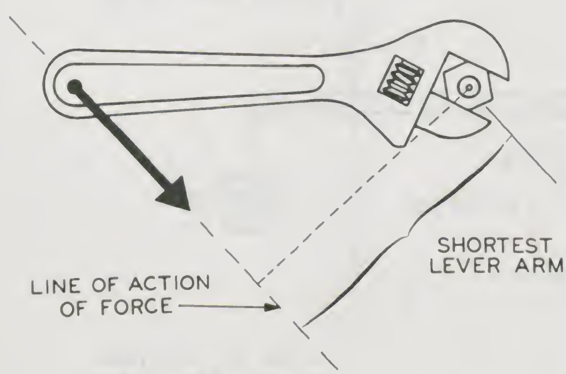
   Item: When the distance from the pan support to the fulcrum of a certain balance is 12 cm, the balance has a sensitivity of 6 div/mg. When this distance is reduced to 9 cm, what is the sensitivity of the balance?

8. *Goal:* Know how to solve problems involving empirical laws which are inverse proportions.

   Item: When the beam center of mass of a certain balance is 0.5 cm below the fulcrum, the balance has a sensitivity of 9 div/mg. When this distance is increased to 1.5 cm, what is the sensitivity of the balance?

Answers for the Items Accompanying the Preceding Goals

1.



2. 0.588 newton-meters; turns the lever counterclockwise.

3. a. 230 newton-meters clockwise.
   b. 180 newton-meters counterclockwise.
   c. 50 newton-meters clockwise; lever will turn clockwise.

4. a. At the center of the hole.
   b. At its center.
   c. Centered at the 50-cm mark.

5. 16.2 newton-meters counterclockwise.

6. $p = 890\,T$

7. 4.5 div/mg

8. 3 div/mg

SECTION B

An Empirical Approach to the Physics Concepts and Principles of the Analytical Balance

THE LEVER

We will re-examine the lever, considering it from a *quantitative* point of view.

A First Class Lever

Figure 14 shows an example of a *first class lever,* where the forces are supplied by the *fulcrum* at A and the weights of objects having masses M_1 and M_2.

Any rotation will occur about the fulcrum. The weight of the body having a mass M_1 would tend to rotate the lever counterclockwise about the fulcrum. The weight of the body having a mass M_2 would tend to rotate the lever clockwise about the fulcrum. The two distances L_1 and L_2 are the *lever arms.* "Lever arm" is the name given to the perpendicular distance from the fulcrum to a *line of action of a force.* The line of action is a line along which a force acts. Each line of action for the lever is shown as a dashed line in Figure 14. Figure 14A shows a first class lever which has different sized masses hanging from it and which is not horizontal.

Problem. What relationship exists between the masses of M_1 and M_2 of the objects and the respective lever arms L_1 and L_2 when the system is in horizontal equilibrium? To solve this problem, you should now do the following experiment.

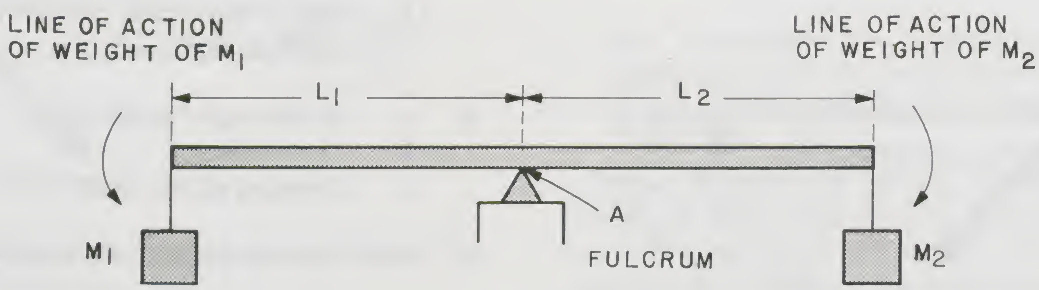


Figure 14.

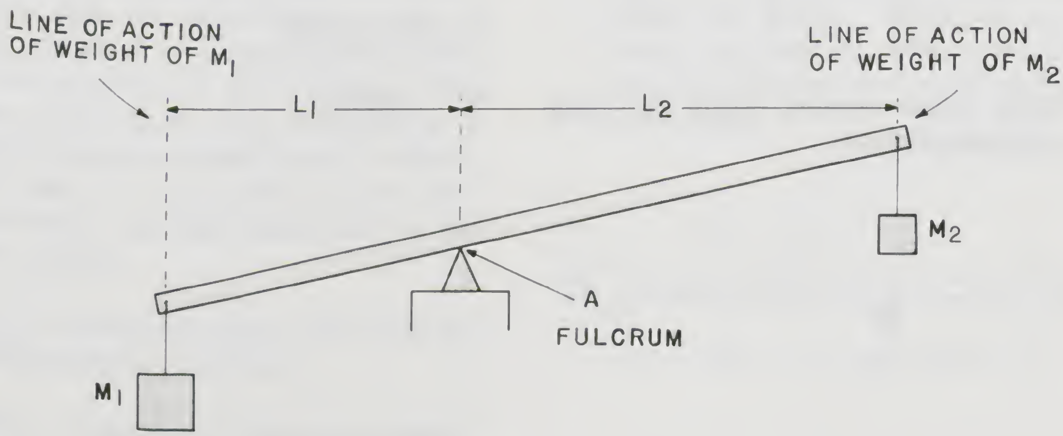


Figure 14A.

EXPERIMENT B-1. Principles of the Lever

You should now take out the work sheets at the end of this module. Write answers to questions, and complete the tables and graphs on those sheets.

For this experiment, we will use the same apparatus you used in Experiment 1 of Section A. You will recall that a meter stick was used as the lever, with an adjustable knife edge and stirrup for the fulcrum. Figure 15 shows this apparatus.

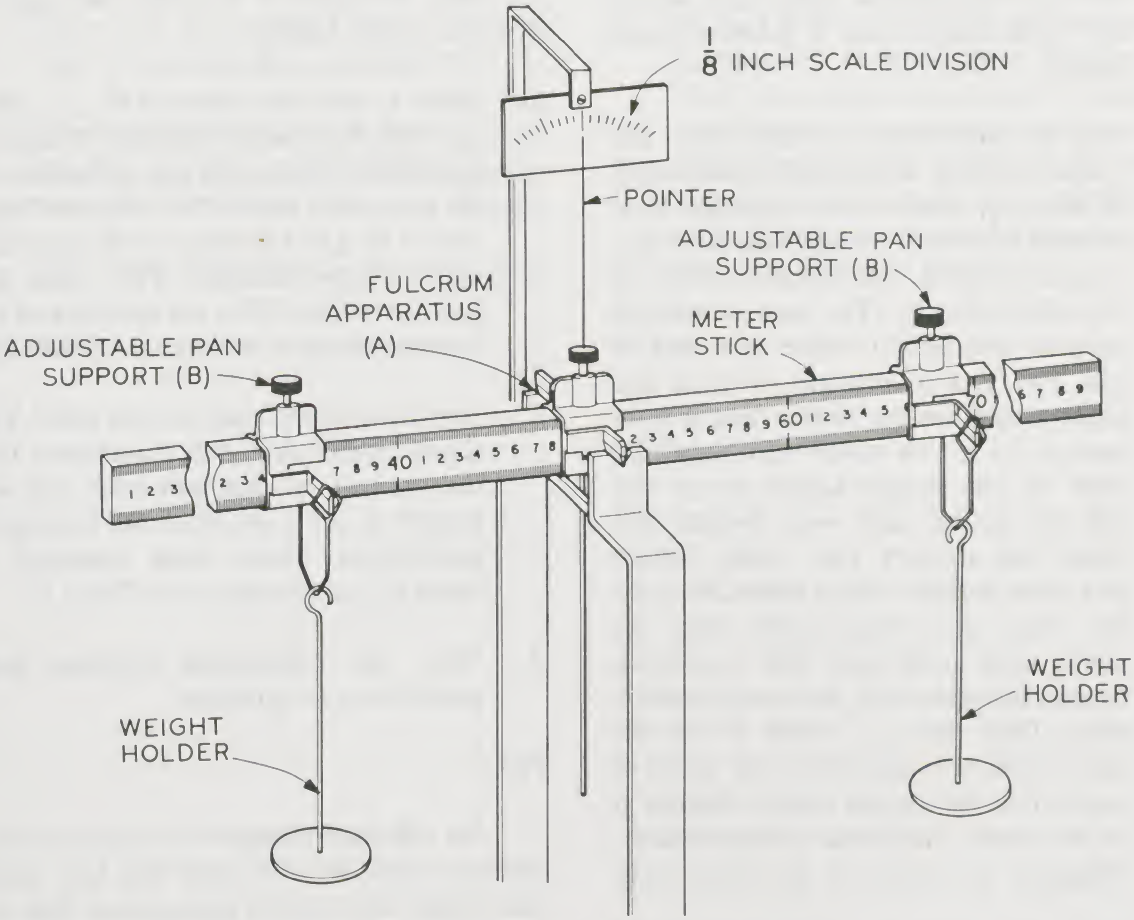


Figure 15.

Part A

1. Remove the adjustable pan supports, B, and the movable beam weights from the lever system. Move the meter stick in the fulcrum apparatus until it is in horizontal equilibrium when placed on the fulcrum. Record the position of the fulcrum (to the nearest 0.1 cm).

2. You are now going to place objects on each end of this lever system and find out how the masses of these objects and their lever arms are related. The adjustable pan supports (B) and weight holders themselves have mass, and we must include these values with those values that are placed on weight holders. Therefore you should now weigh each adjustable pan support on a balance (triple beam).

3. You are now ready to take data. Use Table I on the work sheet where values of M_1, L_1, and M_2 have already been selected. In trial 1 you should place M_1, a total of 100 g, at a distance of 40 cm from the fulcrum. (The mass of the pan support and weight holder are part of this 100 g. For example, if your pan support and weight holder had a total mass of 75 g, you would need only 25 g more on the weight holder to get the 100 g.) In the same way, include the other pan support and weight holder plus some weights which make 200 g for M_2. Then move this weight along the meter stick until you find a position where the system is in horizontal equilibrium. Then find L_2, which is the distance from the fulcrum to the point of support of the weight holder. Record it in the table. Following this procedure, complete the table for the other trials.

4. Study the data in Table I carefully. When L_1 and M_1 are held fixed, how does L_2 change as M_2 is changed? When M_2 is twice as large as M_1, how is L_2 related to L_1?

5. Write an equation which combines all the relationships you found in Step 4.

Part B

Suppose we change the experiment we just did by including a third weight. We will start again by balancing the bare meter stick. Then we will place a weight of mass M_1 on the left side of the fulcrum and weights of masses M_2 and M_3 at different positions on the right side of the fulcrum. You will have to find the mass of the third pan support and weight holder and use that value as part of the total.

1. Find the mass of the third pan support and weight holder.

2. Table II also has values of M_1, L_1, M_2, L_2, and M_3 already selected. In trial 1, you should place 200 g at a distance of 40 cm to the left of the fulcrum. Next, place 150 g at a distance of 40 cm to the right of the fulcrum. Then find the position where 100 g on the right of the fulcrum provides horizontal equilibrium.

3. Now how do we analyze these data? You found in Part A of this experiment that the product of mass and lever arm was needed to get a condition for horizontal equilibrium. Show these products in Table III, using values from Table II.

4. Write the relationship between these products as an equation.

Part C

We will now change the experiment in a different way. We will move the bare meter stick inside the fulcrum apparatus so that it is not balanced to start with. Then we will use two weights, one on each side of the fulcrum, as in Part A, to get horizontal equilibrium.

1. Remove the adjustable pan supports and weight holders from the lever system. Move the meter stick in the fulcrum apparatus until the fulcrum is 10.0 cm from the position where it is in horizontal equilibrium. Record the new position of the fulcrum (to the nearest 0.1 cm).

2. Now add weights to each side of the fulcrum and see what values of masses

and lever arms will give horizontal equilibrium. (Be sure to use the masses of pan supports and wieght holders in your totals, and measure lever arms from the new fulcrum position.) Taking data as before, complete Table IV in the work sheets. Let M_1 and L_1 be on the short end of the meter sticks.

3. How does the product $M_1 L_1$ compare with the product $M_2 L_2$ in each trial?

4. Remove the meter stick completely from the pan supports and fulcrum apparatus, and weigh the meter stick on a balance.

5. You saw previously that the meter stick itself would balance when the fulcrum was at a certain position (Step 1 of Part A). You have moved the fulcrum 10.0 cm from this position of balance. How does the product of the mass of the meter stick and this distance compare with the difference of the products you considered in Step 3?

6. In Part B of this experiment you found a relationship between three weights and their lever arms, when there is horizontal equilibrium. If we assume that the meter stick itself is the third weight, where must all of its weight be considered as acting, if we are to use the equation you found in Step 4 of Part B?

TORQUE

In Part A of Experiment B-1, you studied a lever system with a weight on each side of the fulcrum. You found that when M_2 is doubled, L_2 is cut in half, all else remaining constant. When M_2 was twice M_1, you found that L_2 was half L_1. When M_2 was half as large as M_1, you found that L_2 was twice L_1. The relationship which summarizes these results can be expressed in one of three ways:

$$\frac{M_1}{M_2} = \frac{L_2}{L_1} \quad \text{or} \quad \frac{M_2}{M_1} = \frac{L_1}{L_2} \tag{1a}$$

$$\frac{M_1}{L_2} = \frac{M_2}{L_1} \quad \text{or} \quad \frac{L_2}{M_1} = \frac{L_1}{M_2} \tag{1b}$$

$$M_1 L_1 = M_2 L_2 \tag{1c}$$

Problem 2. Verify, by checking the numbers that each one of these equations applies to the data of Table I of Part A of Experiment B-1 equally well.

Equations (1a) and (1b) are similar to each other in that numbers applying to the situation on opposite sides of the fulcrum are scrambled together; i.e., we have combinations such as M_1/M_2, L_2/L_1, and M_1/M_2 in which quantities with subscripts 1 and 2 are placed together. The form of Equation (1c) is different. The combination $M_1 L_1$ comes from only one side of the fulcrum and the combination $M_2 L_2$ comes from only the opposite side. This means that the combinations $M_1 L_1$ and $M_2 L_2$ are *physical properties* of the lever system on their respective sides of the fulcrum. The meter stick is balanced only if the two quantities are equal. If $M_1 L_1$ is greater than $M_2 L_2$, the stick turns counterclockwise; if $M_1 L_1$ is less than $M_2 L_2$, the stick turns clockwise. The more one of these products exceeds the other, the stronger is the tendency to turn the system in the direction associated with the larger product. We might say that these products measure the strength of a "turning effect."

Calculating Torque

The product ML is a measure of what is called *torque,* or turning effect. But what if the force which pulls on one side of a lever is not a weight of some mass, M? What if we pull on a rope attached to a rod which pivots about some fulcrum. Then we must use the pull in *force* units, as there is no mass M whose weight is producing the force. Also, what if the torque is being produced by some force other than gravitational, like an electrostatic force or a magnetic force? Again, there is no mass whose weight is producing the force.

In Section A of this module, you learned that the weight of an object was proportional to its mass, and that weight on earth could be found from the following equation:

Weight (in newtons) =
9.8 X mass (in kilograms)

This equation can be written with symbols in the form,

$$W = Mg \tag{2}$$

where W is the weight in newtons, M is the mass in kg, and g is a constant having the approximate value on earth of 9.8.

The quantity g is called the *acceleration due to gravity.* It has units of meters per second per second (m/s^2). The acceleration due to gravity is a measure of how much an object speeds up when released and allowed to fall. The unit of 1 meter per second per second multiplied by 1 kilogram is called *1 newton* (N) of force.

Now let us see how to calculate the torque due to the weight of a mass, M. The weight of this mass is:

$$W = Mg$$

The torque due to the weight, W, is defined as the product of the weight and the lever arm.

Therefore,

$$\text{Torque} = WL = (Mg)L \tag{3}$$

For any force, F, the torque is calculated from

$$\text{Torque} = FL \tag{4}$$

Problem 3. A mechanic wishes to tighten a headbolt on an automobile engine to 120 lb·ft of torque. Suppose that he does not have a torque wrench, but does have a spring scale which can be attached to the end of a wrench. If the distance from the center of the bolt to the end of the wrench where the scale is attached is 1.5 ft, with what force must he pull the wrench (at right angles) to produce 120 lb·ft of torque?

Problem 4. How much torque does the weight of a 100-g mass produce if the weight is 30 cm from the fulcrum of a meter stick lever apparatus? Express your answer in torque units of newton-meters (N·m). Take the value of g as 9.8 m/s^2.

WHAT ABOUT THE EQUATIONS FROM EXPERIMENT B-1?

In Experiment B-1 you found, as a condition for horizontal equilibrium, that

$$M_1 L_1 = M_2 L_2$$

We have said that the products ML are measures of torque. What must you do to both sides of this equation to have expressions which are torques? You probably see that we just need to multiply both sides by the constant, g. Then we have

$$M_1 g L_1 = M_2 g L_2$$

or

$$W_1 L_1 = W_2 L_2$$

Problem 5. In Part B of Experiment B-1, you found that when there are three weights, the condition for horizontal equilibrium is

$$M_1 L_1 = M_2 L_2 + M_3 L_3$$

Prove that the torque tending to rotate the lever in one direction is equal to the sum of the torques tending to rotate the lever in the other direction. That is, prove that

$$W_1 L_1 = W_2 L_2 + W_3 L_3$$

HOW TORQUES COMBINE

You now know that the sum of the torques due to the weights on one side of the fulcrum was equal to the torque due to the weight on the other side of the fulcrum. This is a very important result. You have found that the effects of two or more torques acting in the same direction turn out to be *additive.* There was no initial reason to assume that this had to be the case, but we find this simple behavior is an experimental fact.

CENTER OF GRAVITY AND CENTER OF MASS

In Part C of Experiment B-1 you found that, in order to account for all the torque, a certain torque had to be produced by the weight of the meter stick itself. You calculated a measure of this torque by taking the product of the mass of the meter stick and the distance from the fulcrum to the point where the meter stick would balance. We can interpret the results of this experiment in the following way.

Suppose that we imagine a meter stick with no mass or weight, which we will call "weightless." As shown in Figure 16, we can replace the weight and mass of a real meter stick by an object having exactly this weight and mass, and attached at the center of the "weightless" stick (the center of gravity or center of mass).

This imaginary system behaves in exactly the same way as the actual meter stick is observed to behave. The torque equation for

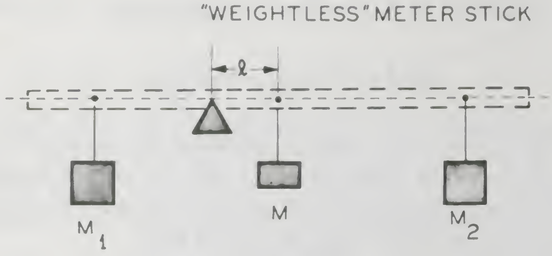


Figure 16.

balance is:

$$M_1 g L_1 = M_2 g L_2 + M g \ell$$

Dividing both sides of this equation by the common factor, g, we have:

$$M_1 L_1 = M_2 L_2 + M\ell \qquad (5)$$

where M is the mass of the meter stick and is the distance from the fulcrum to the center of gravity of the meter stick.

If we realize that the center of gravity of a meter stick is at its center, this example illustrates an important general principle:

> Each particle in a body is attracted to the earth by the force of gravity, but we can always visualize the total pull (the entire weight of the body) as acting (or being concentrated) at a single point called the *center of gravity* of the body.

SOME PROBLEMS AND QUESTIONS

Example Problem

A see-saw is a good example of a first class lever. If the see-saw is 4 m long and its fulcrum is in the center, how far from the center would a boy whose mass is 30 kg have to sit in order to balance a 20-kg boy sitting at the other end?

Solution: Given are

$$M_1 = 30 \text{ kg}, M_2 = 20 \text{ kg} \text{ and } L_2 = 2 \text{ m}$$

In order to solve for L_1, both sides of the equation $M_1 L_1 = M_2 L_2$ may be divided by M_1, giving:

$$L_1 = \frac{M_2 L_2}{M_1}$$

Substituting values into this equation gives:

$$L_1 = \frac{20 \text{ kg} \times 2 \text{ m}}{30 \text{ kg}}$$

or

$$L_1 = \frac{40}{30} \text{ m} = 1.33 \text{ m}$$

Problem 6. How far from the center of a one-meter rod which has a fulcrum at its center must a 0.25-kg mass be placed in order to be in equilibrium with a 0.1-kg mass at the other end of the rod?

Question 10. Is the system shown in Figure 17 in balance? Explain why or why not.

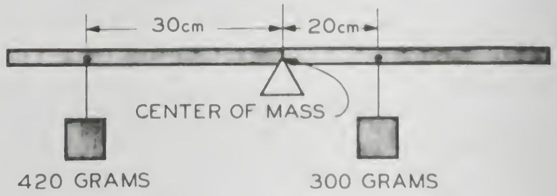


Figure 17.

Question 11. If the system of Figure 17 is not in balance, where would you place the 300-g object to achieve balance, leaving the 420-g object where it is?

Question 12. If the system of Figure 17 is not in balance, and you have the two objects at the indicated positions on the meter stick, which direction (right or left) would you shift the meter stick on the fulcrum to get it balanced? Explain your reasoning.

Question 13. For the system in Figure 17, suppose you had to calculate how far it was

necessary to shift the fulcrum. Do you have sufficient information to make this calculation, or is additional information necessary? If additional information is needed, what would it be?

Problem 7. A see-saw 12 ft long has a weight of 250 lb. Suppose the fulcrum is placed 5 ft from one end and the system is in equilibrium with a child seated on each end. If the child on the short lever arm of the see-saw weighs 80 lb, how much does the other child weigh? Assume that the center of mass of the see-saw is at the midpoint.

WHAT ABOUT TILTED LEVERS?

What if a lever system is tilted at some angle, as shown in Figure 18?

The arm L_1 is no longer perpendicular to the line of action of the weight of the object with mass M_1. Also, the arm L_2 is no longer perpendicular to the line of action of the weights of the object with mass M_2. It is an experimental fact that to account for equilibrium of such a system, the torques must be calculated by using the lever arms, L_1' and L_2'. which are perpendicular to the respective lines of action of weights of objects having masses M_1 and M_2.

EQUILIBRIUM OF BALANCES WHEN BEAM IS NOT HORIZONTAL

An analytical balance often has the beam at some angle with the horizontal, instead of the beam being in a horizontal position. This angle of deflection can be used to measure the small difference between the sample, whose mass is to be found, and the standard masses which have been removed. As a matter of fact, the angular scale at one end of the beam is used to indicate the last decimal place for the measured mass of the sample. This scale can be seen in Figure 19, which shows the

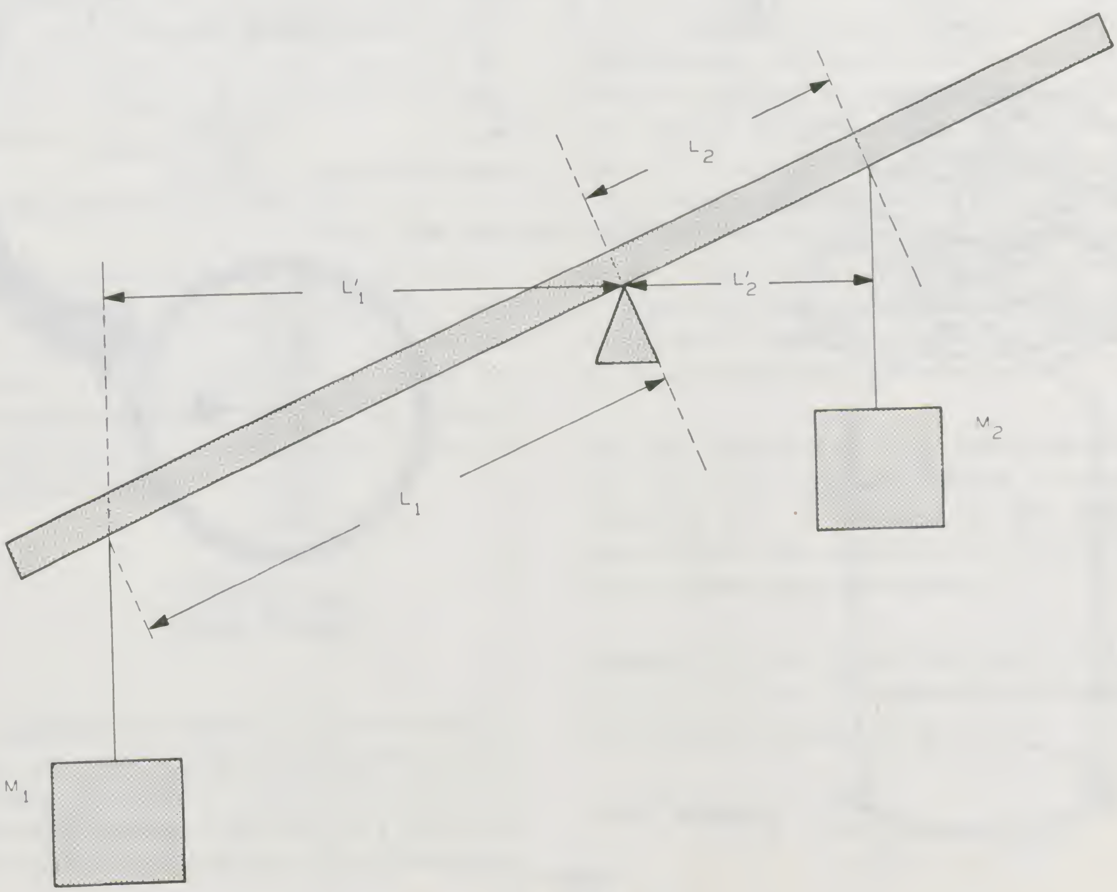


Figure 18.

inside of a typical substitution balance. The tiny beam deflection scale shown is magnified inside the balance and displayed on the front for easy reading. The counterweight in Figure 19 is provided to just balance the system when no sample is in the pan, and all of the removable weights are on the beam. Note that the counterweight is placed below the fulcrum. This increases the stability of the balance. However, it also has other effects which will be investigated later.

CHANGES IN LEVER ARM LENGTHS

When a lever system is not horizontal, the meaning of L_1 and L_2, the lever arms in Equation (5), changes slightly. The correct lever arm values are the distances from the vertical lines through the weights to the vertical line through the fulcrum.

It can be seen from Figure 18 that the lever arms are shorter when the beam is deflected than when the lever is in a horizontal position.

SCALE DIVISIONS AND SENSITIVITY

Since the angular deflection of the beam from a horizontal position is used to measure the small difference between the sample and the standard weights, we want this angular deflection to be as large as possible for a small difference between the known and unknown

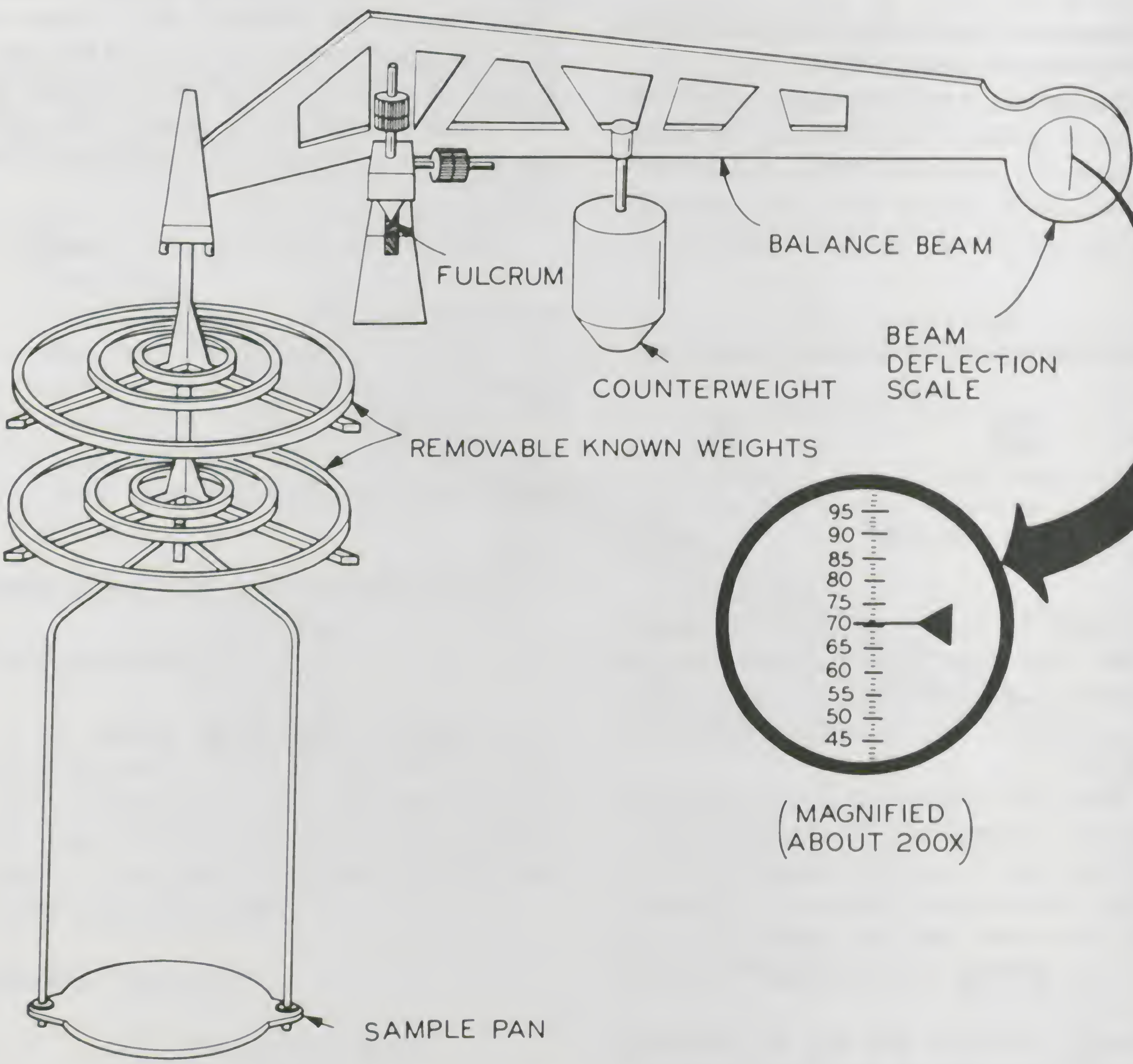


Figure 19.

masses. In fact, the angular deflection caused by a particular small mass difference on the balance beam may be used as a measure of the sensitivity of a balance. Since the angle of beam deflection is quite small for a small excess mass, measuring the angle may be a problem. But, as is indicated in Figure 18, by using a long lever arm, the end of the pointer will go past a large number of divisions on the scale for a small angle of beam deflection. The same situation occurs with the single pan balance. The number of divisions on the beam deflection scale indicates the displacement of the beam from horizontal due to a small excess of unknown mass on the weighing pan. In most instruments, this tiny scale is magnified and projected onto the front of the balance case by a light beam. Notice that the deflection scale on the single pan balance is located out at the end of the balance beam. This gives a large number of divisions on the deflection scale for a small angle of beam deflection. Thus, the angular deflection of the beam of analytical balance from horizontal position is related to the number of divisions shown on a scale and it is the amount of deflection due to a small excess mass that will be used as a measure of the sensitivity of the balance.

DEFINITION OF SENSITIVITY

At least two definitions of sensitivity are used by balance manufacturers. The first defines sensitivity, E, as the ratio of the deflection, s, of the indicating element to the small mass, m_o, producing that deflection. In mathematical form, the definition of balance sensitivity is written

$$E = \frac{s}{m_o} \tag{6}$$

The quantity s in Equation (6) is actually a length on a scale. This length is measured in divisions, the sizes of which are determined by the manufacturer, and which are related to many mechanical features of the instrument. The small mass m_o is usually expressed in milligrams. A milligram is one millionth of the mass of the standard kilogram. The units of sensitivity in Equation (4) would then be "scale divisions per milligram." From here on in this module, the symbol "mg" will be used for milligram and "div" for scale division, which makes the units of sensitivity in Equation (4) div/mg. Very fine balances are made so that even a fraction of a milligram will produce a substantial deflection.

Reciprocal Sensitivity

Manufacturers of analytical balances also used a quantity which is the reciprocal of the sensitivity, E, in Equation (6). Reciprocal sensitivity tells us the value of a scale division, and is given by

$$\text{Reciprocal sensitivity} = \frac{1}{E} = \frac{m_o}{s} \tag{7}$$

The units of reciprocal sensitivity are mg/div. Reciprocal sensitivities for analytical balances range typically from 10 mg/div to 0.5 mg/div. Some manufacturers prefer not to write the units as a ratio, but simply give the number of milligrams needed to produce a deflection of one division. Examples might be "1 div = 10 mg" or "1 div = 0.5 mg." This notation is used primarily for substitution type single pan balances. Many of these balances use a scale on the front of the instrument to further subdivide the value of a division by a factor of ten or even a factor of a hundred. (This scale is called a *vernier scale.*)

Example Problem. A 1-mg displacement on the rider of an equal arm balance produces a deflection of 2.8 divisions on the pointer scale. What is the sensitivity of the balance? What is the reciprocal sensitivity?

Solution. Given in the problem are $s = 2.8$ div and $m_o = 1.0$ mg. The sensitivity of a balance as defined in Equation (6) is

$$E = \frac{s}{m_o}$$

Substituting given values into this equation, we have

$$E = \frac{2.8 \text{ div}}{1.0 \text{ mg}} = 2.8 \frac{\text{div}}{\text{mg}}$$

The reciprocal sensitivity as given in Equation 7 is

$$\frac{1}{E} = \frac{m_o}{s}$$

Substituting given values into this equation we have

$$\frac{1}{E} = \frac{1.0 \text{ mg}}{2.8 \text{ div}}$$

The reciprocal sensitivity is therefore

$$\frac{1}{E} = \frac{0.36 \text{ mg}}{\text{div}}$$

and the value of one division is

$$1 \text{ div} = 0.36 \text{ mg}$$

Problem 8. A 100-mg mass from a standard set of "weights" produces a deflection of 10 divisions on a substitution balance. What is the balance sensitivity and what is the value of 1 division for this balance?

Question 14. Suppose a balance is characterized by the statement 1 div = 0.20 mg. By comparison with values given in preceding examples and discussion, would you describe this as a sensitive or insensitive balance? Why?

We now have a quantitative definition of the sensitivity of a balance. But what factors affect that sensitivity? Our definition of sensitivity makes it appear that all you have to do to increase the sensitivity of a balance is to make the scale divisions smaller, or to lengthen the pointer, so that a given twist of the balance produces a larger deflection. In Experiment A-1 the sensitivity of a balance is increased as the center of mass of the balance is raised closer to the fulcrum. You may now investigate the quantitative dependence of balance sensitivity on this and other variables.

EXPERIMENT B-2.

Sensitivity of a Balance

You should now take out the work sheets at the end of this module. Write answers to questions, and complete the tables and graphs on those sheets.

Part A. Dependence of Balance Sensitivity on Length of Pan Arm

In this part of the experiment, you will use the same apparatus you used in Experiment B-1 and shown in Figure 20 with the adjustable beam mass removed.

1. With the adjustable pan supports, B, removed from both ends of the meter stick, move the fulcrum apparatus, A, to a position near the center of the meter stick where horizontal equilibrium is achieved. Record this position of the fulcrum.

   Next place one of the adjustable pan supports so that it is 45.0 cm to the left of the fulcrum. Add a weight holder and load totaling 150 g. (Don't forget to include the mass of the pan support, as well as the weight holder, in this total.) Place the same load on the other end of the system and adjust the position of that pan support until the balance pointer shows a zero deflection. (Zero deflection means that the pointer falls at the center of the deflection scale.)

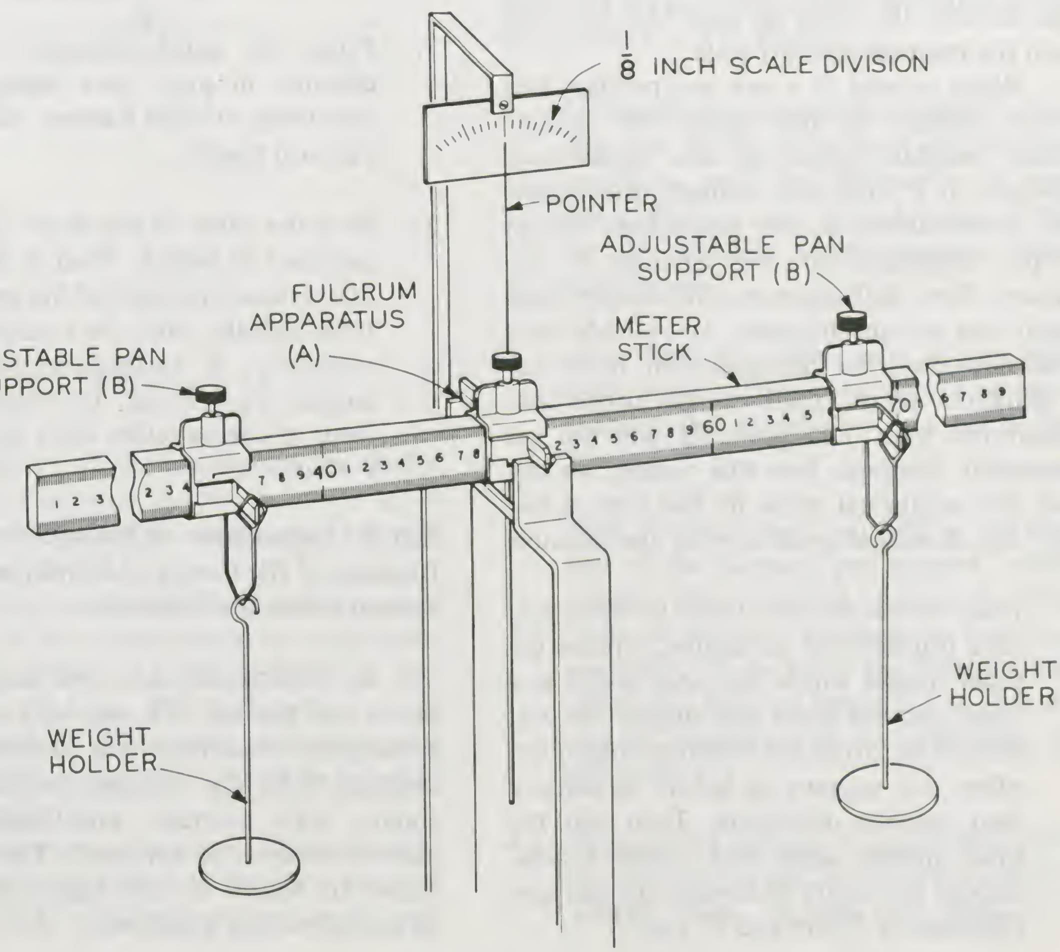


Figure 20.

2. Now, temporarily remove the weight holders and their weights from the system. Weigh the beam, fulcrum apparatus, and pan supports on a triple beam balance.

Put the weight holders and their weights back on the pan supports, place the lever system back on the fulcrum, and balance it again. Place movable weights on the rod below the fulcrum as shown in Figure 23. Use a mass equal to what you just found for the rigid part of the beam system. Position these weights so that the distance from the fulcrum to their center is 10 cm.

With the balance system in equilibrium, add small weights of additional mass to the "unknown" weight holder (let this be the weight holder on the left) and observe the deflections produced on the deflection scale. Begin with a weight having a mass of 2.0 g and increase the mass on each trial by 2.0 g until the position goes off scale.

When moving to a new rest position and before coming to rest again, the pointer should oscillate about its new equilibrium position. If it does not oscillate, the weight you have added is not producing enough torque compared to the friction in the system. New deflection readings under such conditions are questionable. As you add each small mass, look for this oscillation. If you see a situation where there is no oscillation, repeat the trial. You probably will not get consistent readings. For this reason, we say that the additional mass in this case is less than the *threshold sensitivity* of the balance.

3. Record your data in Table V. When you have finished this procedure, remove the small masses which had been added and move the unknown pan support so that it is 35 cm from the fulcrum. Adjust the other pan support as before to achieve zero balance deflection. Then add the small masses again and record results. Repeat the entire procedure for pan arm distances of 25 cm and 15 cm.

You now have data in the table from which you can calculate balance sensitivity. You recall that sensitivity is the deflection s divided by m_o.

4. Calculate values of sensitivity for each trial and enter these calculated values of sensitivity into Table VI.

5. For each of the four pan arm distances (using those trials above threshold) calculate the average balance sensitivity. Convert these values in div/g to div/mg. Use Table VII to show these results.

6. Plot a graph of sensitivity on the vertical axis and pan arm length on the horizontal axis. Use graph paper and draw the best straight line which will pass through these data points and through the origin.

7. From the graph prepared in Step 6, describe in your own words how the sensitivity of this balance changes with pan arm length.

8. Find the value of the slope of the graph prepared in Step 6. What is the value of the vertical intercept of the graph? Using these results write an equation for the sensitivity, E, in terms of the pan arm length, L_p. (Recall the slope-intercept form of the equation for a straight line: $y = mx + b$.)

Part B. Dependence of Balance Sensitivity on Distance of the Center of Gravity of the Beam System below the Fulcrum.

In Experiment A-1, you learned about *center of gravity.* To see why we should investigate sensitivity as a function of the distance from the fulcrum to the center of gravity, let's examine what happens to a balance when it is deflected. The deflection shown in Figure 21 is exaggerated to permit us to analyze the situation.

You can see that point C (at which we visualize all of the mass of the beam system as being concentrated) has been displaced to the right and upward. The weight due to the mass of the balance beam system in this situation produces a torque tending to rotate the system clockwise and back toward horizontal equilibrium, while the weight due to the mass m_0 tends to rotate the system counterclockwise. These two opposing torques must be equal for equilibrium to occur. Otherwise the weight of a small mass would simply rotate a balance until it falls off the fulcrum. To calculate the torque due to the balance beam system, we use the weight due to the mass of the beam, with this weight acting through the lever arm given by the distance in Figure 21.

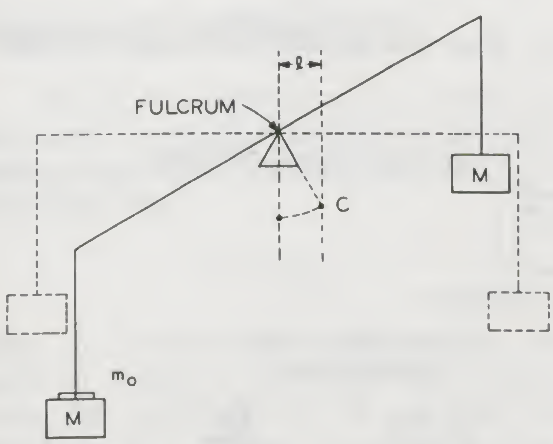


Figure 21.

Now let's find the location of the center of mass of the beam system when it is in equilibrium. Figure 22 shows the balance under that condition.

If we had just a meter stick, the center of gravity of the beam would be approximately at its center, the point A, which is also the fulcrum. We know that the location of the center of mass of the beam is affected by the value of the movable mass, *M*, and its location below the fulcrum.

Suppose that we use a movable mass, *M*, which has the same mass as the total mass of the meter stick, adjustable pan supports, and

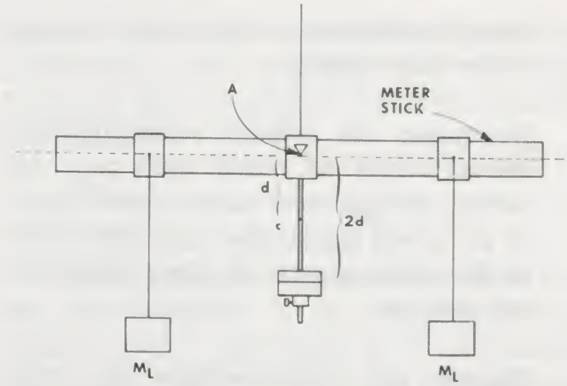


Figure 22.

fulcrum apparatus. With the movable mass removed, the center of mass of the system is very nearly at A. Because the mass of the beam system shown in Figure 22 is evenly split between the movable mass and the rest of the balance, the center of mass of this system is halfway between point A and the center of the movable weight. We have labeled this distance as *d* in Figure 22.

1. Using the same apparatus as before, find the mass of the meter stick, the fulcrum apparatus, and the adjustable pan supports. Place a mass which is equal to this on the movable weight support below the fulcrum.

2. Now find the total mass of the rigid beam system. This total should be twice that of the movable weight below the fulcrum.

3. With a total load of 150 g on each side of the fulcrum (pan support, weight holder, plus weights), adjust the positions of the pan supports so that they are each 45 cm from the fulcrum, and adjust the fulcrum apparatus so that the system balances, with zero deflection on the scale.

4. Measure the distance in centimeters from the fulcrum to the center of the movable weight. Then calculate the distance, *d*,

from the fulcrum to the center of mass of the beam system.

5. Determine the balance sensitivity for several values of added mass, m_o. As before, average these values of sensitivity to get a best single value. Use Table VIII in the work sheets for these data and calculations.

6. Remove any added weights, m_o, and adjust the movable weight to another position. Find the new value of *d.* Then repeat Step 5 to find the sensitivity.

7. Repeat Step 6 for *at least* two additional values of *d.*

8. Prepare a table of values of sensitivity *E* in the left column and values of distance of center of mass of the balance system below the fulcrum, *d*, in the right column.

9. Graph sensitivity, *E*, on the vertical axis, and fulcrum to beam system center of mass distance, *d*, on the horizontal axis. Label the coordinate axes properly and assign values to the axes in such a way that all data points will be included and most of the sheet is used (don't have all points falling in some small region of the paper).

10. From your graph, how does the sensitivity of the balance change as the fulcrum to beam system center of mass distance changes?

11. Since you probably did not get a straight

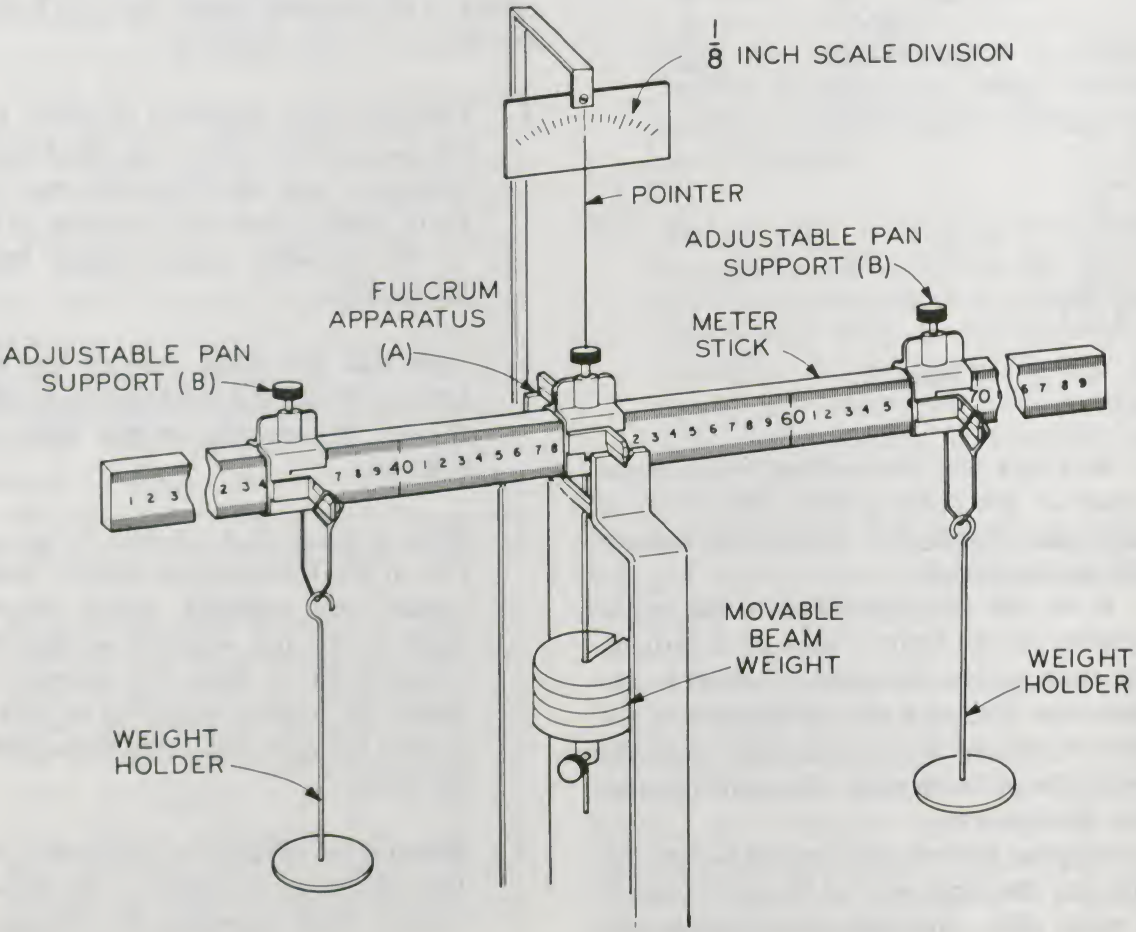


Figure 23.

line, let's try to do something with these results. Prepare another table, using values of E in one column and values of one divided by d ($1/d$) in the other column.

12. Plot a graph of sensitivity, E, on the vertical axis, and $1/d$ on the horizontal axis. Draw the best straight line you can get through these points and the origin (why must the line pass through the origin?).

13. Find the slope and vertical intercept of this straight line, then write an equation relating E and $1/d$.

SUMMARY OF EXPERIMENT B-2

In the experiment you just completed, you found an empirical relationship between balance sensitivity, E, and pan arm length, L_p. You found that

$$E = C_1 L_p$$

where C_1 was the value of the slope of the graph of sensitivity versus pan arm length. From this equation, you can see that the sensitivity of a balance is directly proportional to pan arm length, if everything else is held constant.

You also investigated the relationship of sensitivity, E, to the distance, d, from the fulcrum to the center of mass of the beam system. You found that this relationship could be summarized with the equation,

$$E = \frac{C_2}{d}$$

where C_2 was the value of the slope of the graph of sensitivity versus the reciprocal of the fulcrum to beam center of mass distance, $1/d$. Thus, you can say that the sensitivity of a balance is inversely proportional to the distance from the fulcrum to the center of mass of the beam system, when everything else is held constant.

Question 15. When you did Part A of this experiment, what quantities did you keep constant? What quantities were held constant in Part B?

There are three other variables which we have not examined: 1. the mass, M_b, of the beam system; 2. the length, d, of the pointer; and 3. the total weight, W_T, on the fulcrum.

OTHER VARIABLES

In a given part of Experiment B-2 we should have held all of these variables constant in each experiment, and you held the mass of the beam system constant in each. The total weight on the fulcrum changed only by the amount of the small added weights used to produce deflections for sensitivity calculations. These are very small compared to the total weight of the system, and they would, therefore, not have any measurable effect.

If we did an experiment to determine the balance sensitivity versus mass of the beam system, we would find the relationship:

$$E = \frac{C_3}{M_b}$$

C_3 is the slope of the graph of sensitivity versus the reciprocal of the beam mass. Thus, balance sensitivity is inversely proportional to the mass of the beam system. (This mass does not include the mass of the weight holders or weights on the holders.)

If we keep the scale divisions the same size, sensitivity can be found to be directly proportional to the length of the pointer.

The total weight acting on the fulcrum is a complicated variable to study. The dependence of sensitivity on this variable is related to how the knife edge is deformed under different loads and to the frictional forces involved. Because the load dependence is complicated, substitution balances are de-

signed to keep the load on the fulcrum always the same. In this way, changing loads on the pan cannot change the balance sensitivity.

SUMMARY

Some of the following summarized statements are empirical equations. In Section C of the module we will develop a theory to account for these empirical laws.

A *line of action* of a force is an extended, imaginary line passing through the point of application of a force. The direction of the force can be represented by an arrow which lies on the line of action.

A *lever arm* is the distance from a fulcrum to a line of action of a force.

If M is the mass of a weight hanging from a point on a lever, and L is the lever arm perpendicular to the line of action of this weight, then the product, MgL, is called *torque,* or *turning effect.* Torque can be defined as FL. (For a weight it is WL.)

Torques are *additive.* That is, if three or more weights, having masses M_1, M_2, M_3, etc., are applied on the same side of a fulcrum at points having lever arms L_1, L_2, L_3, etc., perpendicular to the respective lines of action of the weights, then the total torque or turning effect due to weights on that side of the fulcrum is given by the *sum* of the products M_1gL_1, M_2gL_2, M_3gL_3, etc.

Each particle in a body is attracted to the earth by the force of gravity, but we can always visualize the total pull (the entire weight of the body) as acting at a single point: the center of gravity of the body.

When experimental data graph as a straight line passing through the origin, the relationship between the quantity on the vertical axis and that on the horizontal axis is a *direct proportion.* This relationship can be written as an equation in the form

$$V = (\text{slope})H$$

When experimental data graph in the shape of a hyperbola, the relationship between the quantity on the vertical axis, V, and that on the horizontal axis, H, may be an *inverse proportion.* To test for this relationship, V should be plotted against $1/H$. If the result is a straight line passing through the origin, the relationship is an inverse proportion. This relationship can be written as an equation in the form

$$V = \frac{\text{slope}}{H}$$

GOALS FOR SECTION C

The following goals state what you should be able to do after you have completed this section of the module. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item like the one given, you will know that you have met that goal.

1. *Goal:* Understand the derivation of an equation relating physical quantities studied in this section of the module.

   Item: What concepts, laws, or principles are used to deduce the equation

   $$E = \frac{DL_p}{M_{beam}d}$$

   which predicts balance sensitivity? (Answers follow these goals.)

2. *Goal:* Understand the concept of mass density.

   Item: Suppose you are given the following information about the masses and volumes of three different objects:

   a. The volume of object I is twice that of object II.
   b. The mass of object II is three times that of object III.
   c. The mass of object I is one half that of object II.
   d. The volume of object III is equal to that of object I.

   How is the density of object I related to that of object II? How is the density of object III related to that of object I?

3. *Goal:* Know how to make a buoyancy correction for an analytical balance.

   Item: The buoyancy correction equation is given by:

   $$M_s = M_w[\,1+\rho_a(1/\rho_s - 1/\rho_w)\,]$$

   A single pan analytical balance is used to weigh a gold nugget. The standard masses of the balance are made of stainless steel having a density of 8.02 g/cm^3 and gold has a density of 19.3 g/cm^3. If the balance reading is 41.0245 g, what is the mass corrected for buoyancy? $\rho_a = 1.2 \times 10^{-3}$ g/cm^3.

4. *Goal:* Know how to find the arithmetic mean of a set of measurements.

   Item: What is the arithmetic mean of the following measurements of the mass of a sample?

Trial	Measurement
1	2.34 g
2	2.38 g
3	2.34 g
4	2.37 g
5	2.39 g
6	2.34 g

5. *Goal:* Know how to calculate the mean deviation for a set of measurements.

   Item: In the question of Goal 4, the mean of the measurements is 2.36. What is the value of the mean deviation for that set of measurements?

6. *Goal:* Understand the relationship of "best value" to the mean, and "error size" to the mean deviation.

   Item: Two sets of mass measurements are made on the same object, one set using Balance A, the other set using Balance B. For Balance A, the mean is 2.53 and the mean deviation is .04. For Balance B, the mean is 2.58, and the mean deviation is .02. What is the "best value" of each set of measurements? Do these "best values" differ from each other significantly? Why, or why not?

7. *Goal:* Understand the concept of precision.

Item: The following table shows weighings of the same object made on three different balances.

Tell which balance is most precise and why.

TRIAL	BALANCE I	BALANCE II	BALANCE III
1	2.4011	2.4003	2.4012
2	2.4011	2.4005	2.4002
3	2.4011	2.4001	2.4018
4	2.4011	2.4003	2.4008
5	2.4011	2.4002	2.4014
6	2.4011	2.4001	2.4020
7	2.4011	2.4003	2.4004
8	2.4011	2.4005	2.4006
9	2.4011	2.4004	2.4010
10	2.4011	2.4003	2.4016

8. *Goal:* Understand the concept of accuracy.

Item: Refer to the table of measurements in the Item for Goal 7. Suppose Balance III has recently been calibrated with a standard. Are Balances I and II both accurate, or is one accurate and the other not? In the latter case, which is least accurate?

9. *Goal:* Understand the relationship of buoyant force to the density of the fluid involved and the volume of fluid displaced.

Item: Two different objects, A and B, having volumes V_A and V_B, are immersed successively into two different fluids having densities ρ_1 and ρ_2.

Suppose $\rho_2 = 1.5\,\rho_1$

If object A has a buoyant force of 100 g (equal to the weight of 100 g) exerted on it in fluid 1 and object B has a buoyant force of 50 g exerted on it in fluid 2, how does the volume of object A compare with that of object B?

10. *Goal:* Know how to use proportions to solve a problem when two quantities change from one situation to the next.

Item: A balance has a sensitivity of 4.0 div/mg. To improve the sensitivity, the pan arm length is increased by a factor of 16. However, in increasing the pan arm length, the mass of the beam system is increased by a factor of 4. What is the final sensitivity of this balance?

Answers for the Items Accompanying the Preceding Goals

1. The definition of sensitivity, $E = s/m_o$; the condition for balance equilibrium: clockwise torque = counterclockwise torque; concept of torque; concept of center of mass.

2. $\rho_I = (1/4)\rho_{II}$; $\rho_{III} = (2/3)\rho_I$

3. 41.0209 g

4. 2.36

5. 0.02

6. A: Best value is 2.53; B: Best value is 2.58. They do not differ from each other significantly because the ranges of possible values overlap.

7. Balance I is most precise because it has the least spread (or scatter) of measurements.

8. Balance I is accurate, but Balance II is not. The mean of measurements for Balance III, which is known to be accurate, is the same as that of Balance I. The error interval for Balance II is not large enough to overlap with that of Balance III.

9. $V_A = 3\ V_B$

10. 16 div/mg

SECTION C

A Theory of the Analytical Balance; Applications of Concepts and Principles, Errors

EMPIRICAL LAWS

In Section B of this module you found that the sensitivity, E, of a balance was related to the length of the pan arm, L_p, the mass of the beam system, M_{beam}, and the distance, d, from the fulcrum to the center of mass of the beam system. By analyzing graphs which summarized these relationships, you found two equations which related the variables. The third equation was given to you, although you could have also arrived at it experimentally. Those equations are:

$$E = C_1 L_p \qquad (8a)$$

with d and M_{beam} held constant;

$$E = \frac{C_2}{d} \qquad (8b)$$

with L_p and M_{beam} held constant;

$$E = \frac{C_3}{M_{beam}} \qquad (8c)$$

with L_p and d held constant.

A Theory of the Balance

These equations can be called "laws" of the balance. They represent the results of our investigations of what a balance does, but with no explanation as to why a balance behaves this way. We call laws arrived at in such a direct, experimental way *empirical laws.* Empirical laws are essentially numerical summaries or descriptions of experimental facts.

Let us now try to use basic physics concepts and principles of levers to develop a theory of the balance. Then we can see if the theory agrees with the empirical laws we have obtained. If the theory agrees with the facts we have established, we regard it as a useful or "valid" theory. If it does not agree, it would have to be altered or modified. (Of course, the real test of a theory is whether it can be used to correctly predict events which have not yet been observed.)

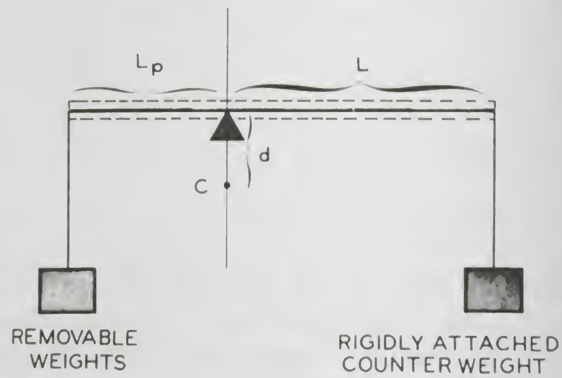


Figure 24.

An Idealized Single Pan Balance

Let us start by considering a single pan substitution balance in horizontal equilibrium with no sample, as shown schematically in Figure 24. The balance consists of a beam, a rigidly attached counterweight on the right, and the suspended pan on the left. The pan load includes removable weights, so that the total weight acting on the fulcrum is always constant. As you weigh an object, you remove weights equal to the weight of the unknown. The center of mass of the beam system is located at point C.

Torques Acting on a System

A small excess mass, m_o, on the pan produces a small angular deflection of the beam. This deflection is exaggerated in Figure 25.

In effect, what is happening is that the torque due to the weight of the small mass, m_o, tends to turn the system counterclockwise, while the torque due to the weight of the beam system mass (including the rigidly attached counterweight), acting at the center

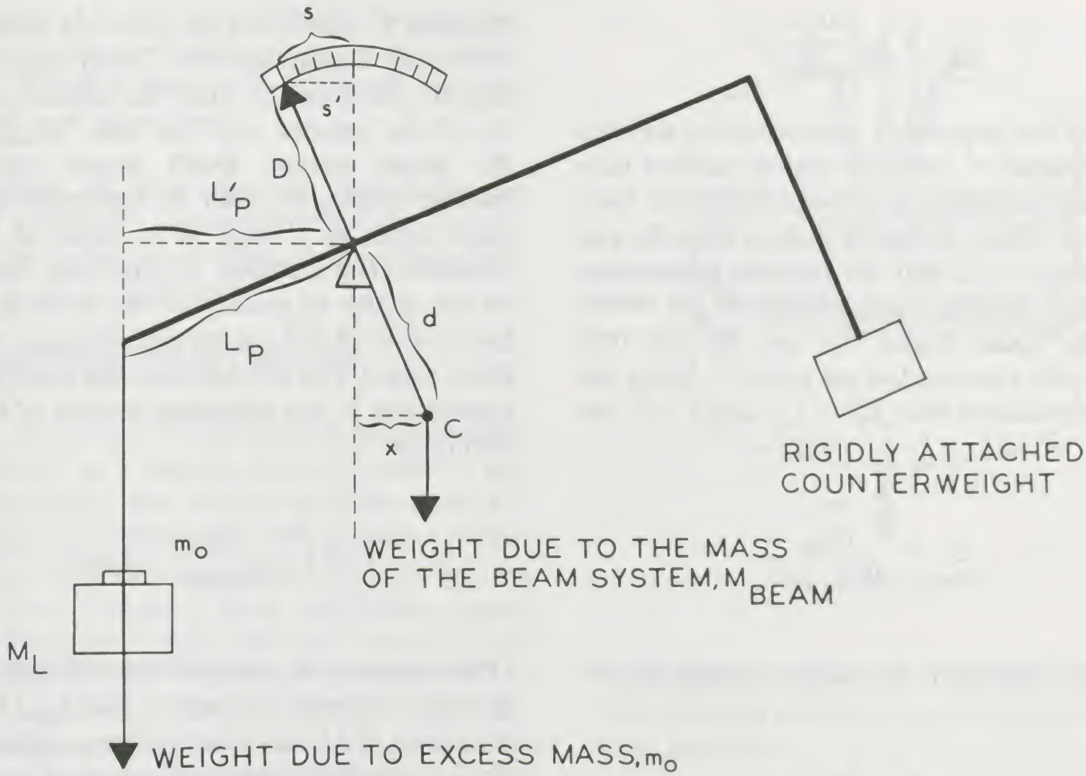


Figure 25.

of gravity, tends to turn the system clockwise. When the system comes to rest at a certain deflection, *s,* the two torques are equal. The lever arm for the weight of m_o is L'_p as shown in Figure 25. The lever arm for the weight of the beam system is x in the figure. The counterclockwise torque is therefore $m_o g L p'$ and the clockwise torque is $M_{beam} g x$, so that at equilibrium

$$m_o L'_p = M_{beam} g x$$

dividing both sides of this equation by g we have:

$$m_o L'_p = M_{beam} x \qquad (9)$$

THE DERIVATION OF A THEORETICAL EQUATION

In Figure 25, the triangle formed by the pointer, D, the distance, s', and the vertical line through the fulcrum, has the same angles as the triangle formed by d, x, and the vertical line. These triangles are therefore *similar,* and we can set up the proportion

$$\frac{s'}{D} = \frac{x}{d}$$

Question 16. In this equation, both s' and D should be measured in units of scale divisions. Why is this the case?

Solving this equation for x, we have:

$$x = \frac{s'd}{D}$$

Using this value of x in Equation (9), we have:

$$m_o L_p = M_{beam}\,(s'd/D)$$

Multiplying both members of this equation by D and then dividing both members by $M_{beam} d m_o$ we have

$$\frac{s'}{m_o} = \frac{DL_p}{M_{beam}d}$$

In the expression derived so far, we have the distance s' which is slightly shorter than the scale distance s. We also have the lever arm L'_p, which is slightly shorter than the pan arm length, L_p. But we measure sensitivities for small angular displacements of the beam, and for small angles, L'_p and L_p are very nearly the same and so are s and s'. Using the approximations that $L'_p \doteq L_p$ and $s = s'$ we have:

$$\frac{s}{m_o} = \frac{DL_p}{M_{beam}d}$$

But our definition for balance sensitivity is:

$$E = \frac{s}{m_o}$$

Therefore, we have:

$$E = \frac{DL_p}{M_{beam}d} \qquad (10)$$

We have used the simple theory of levers to derive this equation. We have, by our approximations, ignored the effect of shortening the lever arm by the angular deflection, and we have not taken the functional properties of the fulcrum into account. Thus do we not expect the equation to give all of the dependences correctly.

COMPARING THEORETICAL PREDICTIONS WITH EMPIRICAL RESULTS

Question 17. Show that if L_p and d in Equation (10) are expressed in cm, and M_{beam} is in grams, then the sensitivity, E, has the correct units only if D is expressed in length units which are the size of the balance scale divisions.

Problem 9. Refer to your data and results in Part A of Experiment B-2. From these data, find the distance, D, from the fulcrum to the tip of the pointer, and the mass, M_{beam}, of the beam system. Don't forget that the pointer length, D, must be expressed in the same units as s, that is in units of scale divisions. The distance, d, from the fulcrum to the center of gravity of the beam system has a value of 5.0 cm for this situation. Using these values, you can calculate the quantity in parentheses in the following version of Equation (10).

$$E = \left(\frac{D}{M_{beam}d} \right) L_p$$

(The quantity in parentheses is the proportionality constant between E and L_p.) Using the values of 15 cm, 25 cm, 35 cm, and 45 cm for L_p, calculate theoretical values of sensitivity E for each value of L_p. Plot these values of E against L_p on the same graph that you used in Part A of Experiment B-2. How does this theoretical graph compare with the empirical graph you had in Part A?

Problem 10. Refer to your data and results in Part B of Experiment B-2. Using measured values of D (in units of scale division), L_p, and M_{beam}, calculate the quantity in parentheses:

$$E = \frac{(DL_p/M_{beam})}{d}$$

Using the values of d you had in Part B of Experiment B-2, calculate corresponding values of sensitivity. How does this theoretical graph compare with the empirical graph you had in Part B? Plot E versus $1/d$ on the same graph you plotted previously.

LIMITATIONS ON BALANCE SENSITIVITY

From Equation 10 you can draw some conclusions about limitations on balance sensitivity.

Question 18. For maximum sensitivity with a given location of center of mass, how should you design a balance in terms of pan arm length and beam mass?

Question 19. How can you increase the pan arm length of a balance without also increasing the beam mass? What design characteristics must you include?

DAMPING OF OSCILLATIONS

As you saw in Experiment B-2, the beam assembly of a balance does not come to rest immediately, but instead oscillates about the equilibrium position. If the frictional effects at the knife edge were very small, as they are for good balances, these oscillations would continue for a long time and wouldn't want to wait, on each weighing, for the oscillations to "run down." When an object–for example, a swing–oscillates back and forth, each swing goes a little less far than the one before. We say that the oscillation is *damped.* The greater the damping, the sooner the swinging slows to a stop, with fewer swings. In a balance, we wish to damp the oscillations, without reducing the balance sensitivity. For this purpose, various damping (or braking) mechanisms are used.

A common type of device is an air damping device such as that shown in Figure 26. A piston is attached to the beam. The piston moves in a cylinder, and frictional effects due to the flow of air into and out of the cylinder cause the desired braking action.

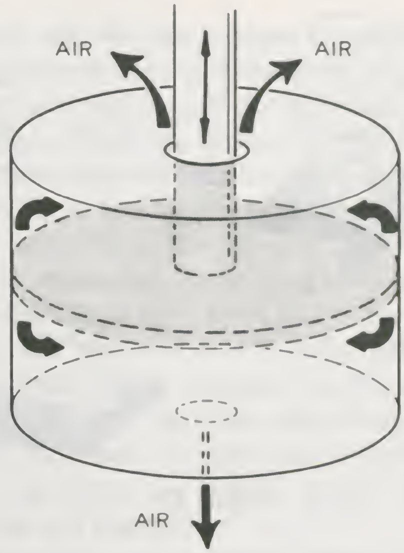


Figure 26.

Question 20. Invent one or two other damping mechanisms that might have an effect similar to the device shown in Figure 26, but which do not involve a piston and a fluid.

Question 21. In Experiment B-2, you may have observed that the oscillation rate changed as the pan arm lengths were changed and as the distance between the fulcrum and the center of mass of the beam system changed. How did these oscillation rates change? Why do you believe that they change in the ways in which they do? (Compare the beam system with other cases of oscillation that are more familiar to you in order to assist your argument.)

THE FULCRUM OF A BALANCE

The most critical components of any balance are the knife edges and planes (or bearing stones) which make up the fulcrum. The pan is also suspended from a knife edge to assure that it remains hanging vertically downward. To obtain a free swing of the beam and the pan, unhindered by friction, the knife edges must be made as small as possible. No edge can be made infinitely sharp. A good balance knife edge might have a radius of curvature as small as 2×10^{-7} m or .00001 in. The radius of curvature is the radius of a circle which could be constructed that would include the knife edge as part of its arc.

STRESSES AT THE FULCRUM

The beam knife edge must support, on a very small surface area, the maximum load of the instrument plus the weight of the beam system. This load produces extremely high contact stresses on the knife edge and plane. Stress (or pressure) is defined as force per unit area, or

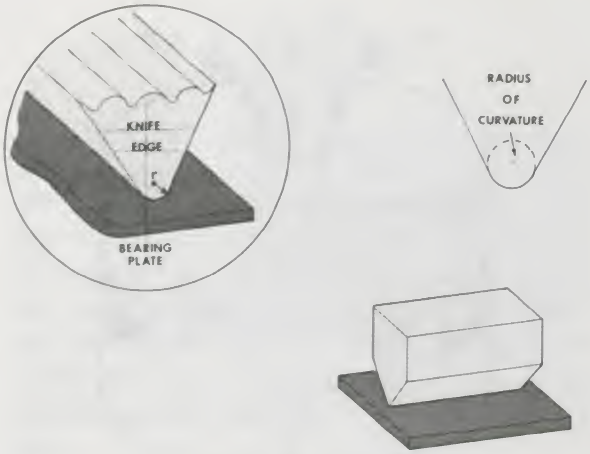


Figure 27.

$$\text{Stress} = \frac{\text{force}}{\text{contact area}}$$

Actual knife edge contact stresses in balances may reach tens of thousands of pounds per square inch. Of course high pressures do not mean that the total load is high. You can push down on a sheet of paper with a sharp pencil and produce large impressions.

Problem 11. Suppose that you push a sharp pencil down on a pad of paper with a force of 4 lb. If the pencil lead is touching a contact area of 0.0015 square inches (an area of 0.0015 in^2 is about 1 mm^2), how much pressure is the pencil lead applying to the area of contact on the paper?

These pressures necessitate using very hard substances for the knife edges and planes in order to prevent permanent deformation of the surfaces and consequent inaccuracies. The substance primarily used in current manufacture is synthetic sapphire, which is a laboratory grown crystal of very pure aluminum oxide (Al_2O_3). Next to diamond, sapphire is the hardest known mineral and is therefore particularly suitable for the purpose. But even though a hard substance can withstand the loads of weighing, a mechanical shock (dropping or jolting) may easily break these critical pieces. To prevent this from happening, the beam and pan are automatically lifted each time the balance is stopped, so that no contact is made between the knife edge and the plane except when readings are being taken. Damage to the knife edge is probably the main factor which limits the useful life of a balance.

SOURCES OF ERROR

Deformations and Load

Deformation of the contact areas has some small effect upon the sensitivity of a balance. Errors due to such effects are eliminated in the substitution balance (the single pan type). In the substitution balance the load on the balance beam and knife edges at equilibrium is always the same regardless of the amount weighed. Therefore, deformations are nearly the same for all weighings. Sensitivity, therefore, does not depend on the load placed on the pan in this type of instrument.

Removable Weights as Standards

A possible source of error, not often considered by the user of a balance, is the set of removable weights included in the balance. These are very carefully produced copies of standard weights. Indirectly they are copies of the standard kilogram or its subunits. Typically, the weights may be in the form of rings which are resting on the pan arm above the weighing compartment of the balance. They are removed mechanically as the sample is made. Although these weights are quite accurately made, their mass is subject to change, over a period of years, due to wear or corrosion. To minimize this problem, the weights are usually made of tough and corrosion-resistant steel.

Other Sources of Balance Error

Static electricity can also contribute significantly to weighing errors. When a piece of glass, plastic, or rubber is rubbed with a cloth, it attracts (and is attracted by) nearby objects. This effect is called "electrostatic charging." Such effects are not uncommon with glass or ceramic containers, although the resulting forces are usually not noticeable. However, when a person is weighing a sample

to the nearest 0.1 mg, these electrostatic forces can be significant. To avoid such error, one must allow a sufficient length of time, after wiping a vessel to be weighed or the glass of the balance itself, before starting the weighing operation. Sometimes a small radioactive source is placed in the weighing compartment to reduce the electrostatic problem. This radioactive source ionizes the air (causes the neutral atoms to break apart into charged particles), and these charges provide a path by which other charges on nearby objects can leak away.

Buoyancy

Suppose we weigh an object in air and find that it produces a balance reading of 16 g; then while it is still suspended from our balance, we submerge the object in water and find that the balance reading is now 14 g. We know that the mass of the object has not changed. Yet the mass reading on the balance is less. We can account for this effect by remembering that it is the force of gravity on the object which pulls down on the pan of the balance. When the object is submerged in water, there must be another force acting on the object, a force due to the water. For objects which sink, this upward force is less than the downward force of gravity on the object. But the upward force due to the water reduces the net downward pull, making a measurement of the weight of the object smaller. We know that the pull of gravity is not any less just because the object is in water; therefore, the actual weight of the object has not changed.

Whenever the measured weight of an object changes in this way, when we know that its mass and actual weight have not changed, we refer to its measured weight as its *apparent weight.*

The upward force we have been discussing is called a *buoyant* force. To investigate buoyant forces, you should now complete Experiment C-1, Archimedes' Principle.

EXPERIMENT C-1. Archimedes' Principle

Write answers to questions, and complete the table on the work sheet at the back of this module.

You have been given sample objects in the form of cylinders.

1. Measure the diameter d and length L of each sample. Record these measurements on the chart in the work sheets.

2. Using the formula,

$$V_s = \frac{\pi}{4} d^2 L$$

calculate the volume of each sample. Enter these results in column 3 of Table I in the work sheets.

3. Find the mass reading for a sample in air; then suspend the sample by means of a thread from the hook on the balance as shown in Figure 28, and thereby find the mass reading when the sample is submerged in water. For this sample, record your data in row 1, columns 1 and 2 of Table I.

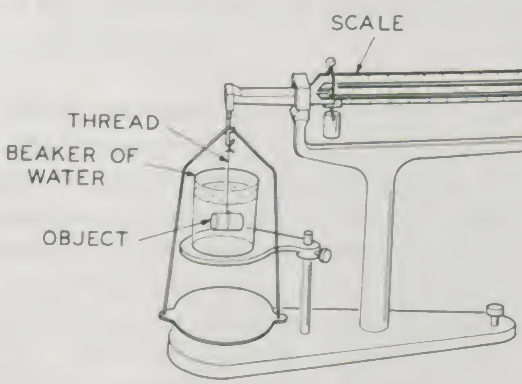


Figure 28.

4. Repeat Step 3 for the other four samples.

In Experiment A-1 we referred to materials which were "heavy" and others which were "light." Let's try to define this concept more carefully.

Suppose we have a mass M of some material and find that the volume of the sample is V. Consider the number we obtain by making the calculation

$$\frac{M \text{ (in g)}}{V \text{ (in cm}^3\text{)}}$$

We have calculated the number of grams (mass) associated with each cubic centimeter of the material. This number is a property of any particular substance, and we give it the name *density*. If this number is large (as it is for mercury, 13.6 g/cm^3), we say that the substance is very dense. If the number is small (as it is for air under ordinary conditions, about 1.2×10^{-3} g/cm^3), we say that the substance has a low density.

If we assign the symbol ρ to density, then we have the statement

$$\rho = \frac{M}{V}$$

Example Problem. Determine the density of a sample which has a mass of 800 g and a volume of 100 cm^3.

Solution. Given are:

$$M = 800 \text{ g and } V = 100 \text{ cm}^3$$

Substitution into the definition $\rho = M/V$ gives

$$\rho = \frac{800 \text{ g}}{100 \text{ cm}^3} = 8 \text{ g/cm}^3$$

This is the density of stainless steel.

Problem 12. Determine the density of a 150-mg (0.150 g) sample if its volume is 0.05 cm^3.

Let us use this definition to find the numerical value of the density of water.

5. Measure the mass of some known volume of water, say 100 cm^3, and calculate the density.

   Once we know the density of a material (mass of 1 cm^3), it is easy to calculate the total mass of any volume we might be interested in,

$$M = \rho V$$

   Use this idea to calculate the mass of water displaced in the experiment you performed with the cylinders.

6. Fill in the last column (number 5) of Table I by using the volumes listed in column number 3 and the value of the density of water that you have just obtained.

7. Now compare the values you have listed in columns 4 and 5. Although these measurements have been expressed in terms of mass units (grams, in this case), we know that objects having these masses also have certain weights. We could calculate these weights by multiplying each mass by the acceleration due to gravity. Thus you can consider column 4 to represent an upward force on the object. For the same reason, column 5 can represent the weight of the water displaced. What conclusion do the results of your experiment point to regarding the connection between the upward force water exerts on a submerged body and the weight of the water this body displaces (or pushes aside)?

Question 22. When we perform the usual operation of weighing an object on our balance, the object is not submerged in water, but it is submerged in air. And air exerts an upward, buoyant force just as water does. Would you expect the buoyant force exerted by air to be large or small? Explain your reasoning clearly and carefully.

Question 23. The buoyant force does affect the measured weight of an object. Why does the buoyant force also affect the measured mass of an object?

Question 24. Under what circumstances might the buoyant effect of the air be large enough to make your balance measurement differ significantly from the true mass: for an object with a large mass and a small volume or an object with a small mass and a large volume?

ARCHIMEDES' PRINCIPLE

In Experiment C-1, you showed empirically that the upward (buoyant) force on an object submerged in water is equal to the weight of the water displaced. This is true in general, for objects submerged in any fluid, and this statement is known as "Archimedes' Principle."

If we think about the situation a little bit, the statement makes perfectly good sense and we might even have been able to predict it (as Archimedes did) without actually having performed the experiment.

Buoyant Force of Water on Water

Consider a container of water, as shown in Figure 29, and let us focus our attention on a glob of water of any shape, like the one outlined.

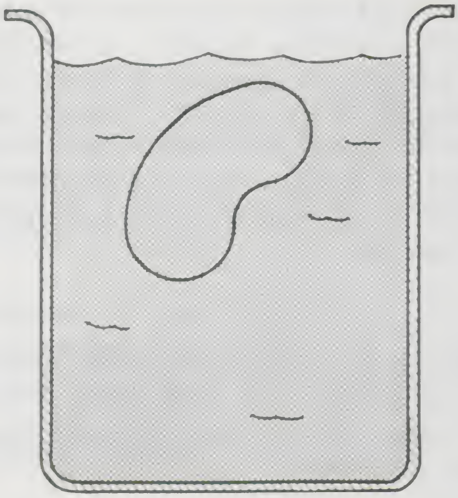


Figure 29.

This chunk of water is being pulled by the earth's gravity (has its own weight W_w), and, if it were taken out of the container and let go, it would fall to the ground.

Inside the container, however, this chunk of water sits still in its outlined location, and neither rises nor falls. We are therefore forced to conclude that our outlined glob is being held up by the surrounding water (that which is outside the outlined shape), and that the upward force exerted by the surrounding water is exactly equal to the weight of the glob. Let us show this on a simple diagram in which the forces we are describing are represented by arrows (Figure 30). The weight of water is represented by W_w and the buoyant force by B.

Because the chunk of water neither rises nor falls, we must conclude that the forces acting on it are balanced, and that

$$B = W_w$$

The statement we have made is true, of course, of any and every chunk of water in the container, regardless of location, size, or shape.

Buoyant Force of Water on Other Substances

Now suppose that we submerge into our container a chunk of some other material, a chunk having exactly the same size and shape as our glob of water, but having its own weight W_m.

We now have a situation in which a particular glob of water has been pushed aside and replaced by a different chunk of material. For a glob of given size and shape, the surrounding water exerted a particular upward force B (exactly equal to the weight of the glob of water), and we expect it to keep exerting exactly this same force on the new chunk of material we have inserted. This force is shown in Figure 31.

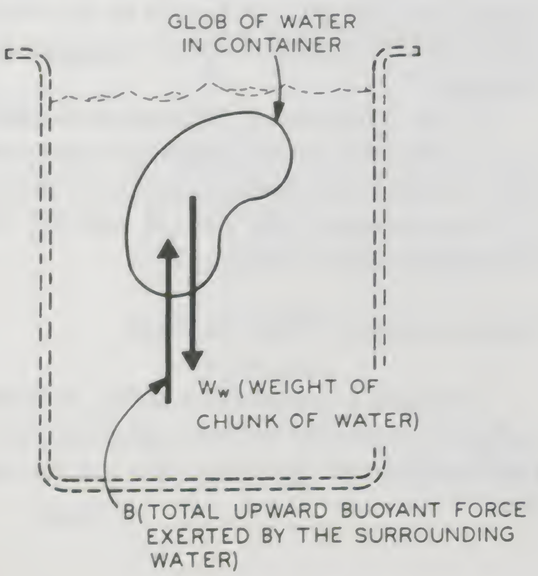


Figure 30.

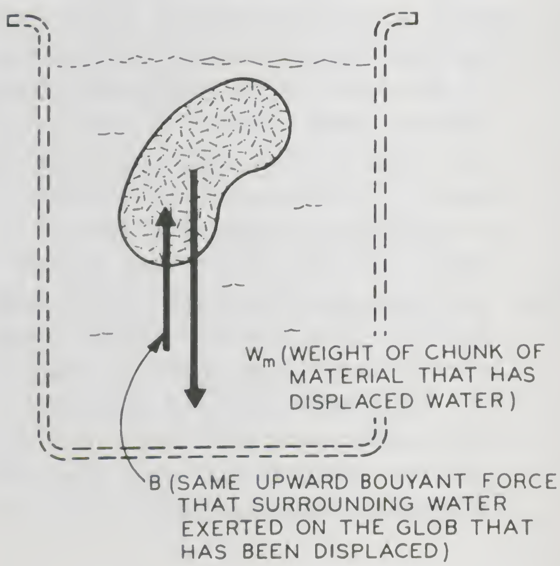


Figure 31.

In other words, we have arrived at the conclusion that any object we submerge in water will be subjected to an upward force exactly equal to the weight of the water that has been displaced. We have arrived at Archimedes' Principle.

Question 25. If, in Figure 31, the weight W_m of the new chunk of material is greater than the weight B of water displaced, what will happen to the chunk if we let it go? How does the density, ρ_m, of the material compare with the density, ρ_w, of water in this case?

Question 26. If in Figure 31 the submerged chunk of material rises when you let it go, what can you say about the relative sizes of the arrows representing W_m and B? How does the density of the material compare with that of water?

Question 27. Describe in your own words the condition under which a piece of wood floats on water. How much of it must be submerged for the piece of wood to float?

Question 28. Suppose the surrounding fluid in Figure 31 were air instead of water, but the submerged material were the same. How would the size of B compare with W_m?

Buoyant Force in Terms of Density of Fluid

In the case of buoyant force, we can express the force in terms of the density and volume of the fluid which is displaced. Since the weight on an object can be taken as its mass times the acceleration of gravity, we can write the buoyant force as

$$B = M_d g$$

where M_d is the mass of fluid displaced and g is the acceleration due to gravity. But using the definition of density, we can write the mass as

$$M_d = \rho_f V_d \qquad (11)$$

where ρ_f is the density of the fluid and V_d is the volume of the fluid which is displaced by the object. Then the buoyant force can be expressed in terms of the density and volume of the fluid.

$$\text{Buoyant force} = \rho_f V_d g \qquad (12)$$

If we are interested in the mass of fluid displaced, we need only calculate the product $\rho_f V_d$. When the object is completely submerged in the fluid, the volume V_d is the same as the volume of the object.

Example Problem. What is the buoyant force in air on a helium filled balloon which has a mass of 3 g and a volume of 5000 cm^3? Use the mass of fluid displaced as the measure of buoyant force. The density of air is 1.2×10^{-3} g/cm^3. Will the balloon rise or fall in still air?

Solution. Given are:

$$M = 3 \text{ g},\ V_d = 5000 \text{ cm}^3$$
$$\text{and } \rho_f = 1.2 \times 10^{-3} \text{ g/cm}^3$$

Substituting values of V_d and ρ_f into Equation (11), we obtain the buoyant force in terms of the mass of the displaced air,

$$M_d = (1.2 \times 10^{-3} \text{g/cm}^2)(5 \times 10^3 \text{cm}^3)$$

or

$$M_d = 6\text{g}$$

Since this measure of buoyant force is greater than the mass of the balloon, the balloon will rise in the air.

Problem 13. What is the buoyant force on a helium balloon which is in the air and which has a volume of 7×10^7 cm^3? Express the buoyant force in terms of the mass of air displaced. Will it lift a 170-lb (7.7×10^4 g) man?

Problem 14. A nearly vibration-free table may be constructed by floating a steel slab in a container of liquid mercury. A 13-kg slab displaces approximately 1000 cm^3 (10^{-3} m^3)

of mercury. Using these figures, what is the density of mercury? Is the volume of the slab larger or smaller than 1000 cm^3?

BUOYANCY CORRECTION FOR THE ANALYTICAL BALANCE

To correct for buoyant force when using an analytical balance, one must know the density of air. The density (ρ_a) of air at atmospheric pressure and room temperature is about 1.2×10^{-3} g/cm^3. Using this value and the buoyancy equation, we can derive a correction factor for weighing samples in air. In the following derivation, we shall correct only that part of the sample weight which is balanced by removing the weight rings. We might, in principle, also apply a similar correction to the part of the weight read from the beam angle (fractions of a gram), but this correction is far too small to be significant.

Suppose we have a sample on a balance pan as shown in Figure 32.

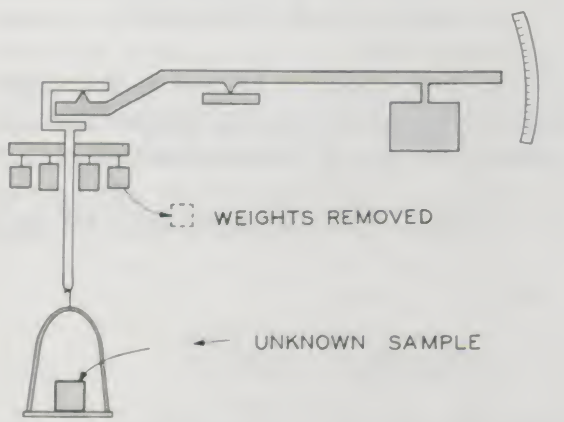


Figure 32.

In the absence of any buoyant forces, the situation is as shown in Figure 33.

The force of gravity W_w on the removed weights is equal to the force of gravity W_s on the unknown sample, and the system is in balance as it was before the unknown was placed on the pan:

$$W_w = W_s$$

In the presence of buoyant forces from the surrounding air, the situation becomes slightly different, as indicated in Figure 34.

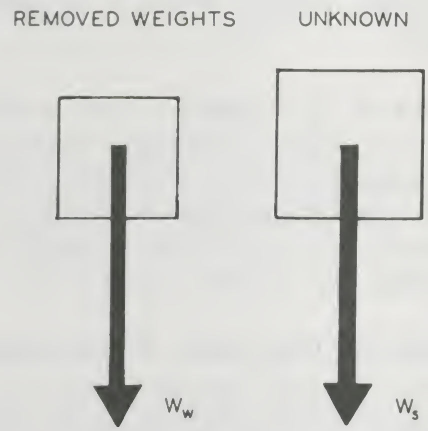


Figure 33.

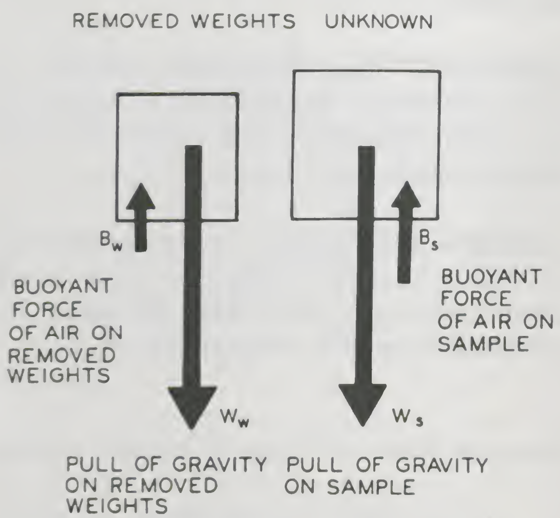


Figure 34.

The net downward force exerted by the removed weights on the balance arm must have been

$$W_w - B_w$$

while the net downward force being exerted on the balance arm by the sample must be

$$W_s - B_s$$

Thus, when the system is rebalanced, we must have the equation

$$W_w - B_w = W_s - B_s \tag{13}$$

where W_w and W_s are the true values of the removed weights and the sample, respectively, and B_w and B_s are the weights of air displaced by the removable weights and the sample, respectively.

We could also write this as

$$M_w g - M_{aw} g = M_s g - M_{as} g$$

where M_{aw} is the mass of the air which is displaced by the removed weights and M_{as} is the mass of the air which is displaced by the sample.

If we now divide both sides of this equation by the common factor g, we have

$$M_w - M_{aw} = M_s - M_{as} \tag{14}$$

If we denote the volumes of the removed weights and the sample by V_w and V_s and the density of the air by ρ_a, we can use the definition of density to write this in the form

$$M_w - \rho_a V_w = M_s - \rho_a V_s \tag{15}$$

Question 29. Explain in your own words how Equation (15) follows from Equation (14).

By using the definition $\rho = M/V$ in the form $V = M/\rho$, we may also express the volumes V_w and V_s in terms of the density of the weights, ρ_w, and the density of the sample, ρ_s. Then we have

$$M_w - \rho_a(M_w/\rho_w) = M_s - \rho_a(M_s/\rho_s)$$

Now we can factor out the mass M_w on the left and factor out the mass M_s on the right,

$$M_w(1 - \rho_a/\rho_w) = M_s(1 - \rho_a/\rho_s)$$

It is now possible to solve for M_s, the actual mass of the sample, in terms of M_w, the mass of the removed weights which is indicated by the balance reading:

$$M_s = M_w \frac{(1-\rho_a/\rho_w)}{(1-\rho_a/\rho_s)} \tag{16}$$

THE CORRECTION FORMULA

Equation (16) represents a solution to the problem we have posed, but it is certainly not an equation that is very easy to use. Both the ratios ρ_a/ρ_w and ρ_a/ρ_s are likely to be quite small. As a matter of fact, if the weights are stainless steel, then $\rho_w = 8.02$ g/cm^3 and $\rho_a/\rho_w = 1.2 \times 10^{-3}/8.02 = 1.5 \times 10^{-4}$. The ratio ρ_a/ρ_s may be of about the same order of magnitude. These small values mean that in order to find the true mass from the indicated mass by using this equation, we must multiply by a ratio of two numbers, each of which is very slightly less than one. In a slide rule calculation, this ratio may well be indistinguishable from the number one. For this reason, we usually need a little more algebra to express our correction equation in a more usable form.

In Equation (16) we are dividing by a number very slightly less than one, $(1-\rho_a/\rho_s)$, or alternatively we could say we have as a factor, $1/(1 - \rho_a/\rho_s)$. Dividing the number one by something very slightly less than one should give an answer slightly more than one. For such a division, the answer is actually about the same amount greater than one as the divisor is smaller than one.

For example:

$$1/.999 = 1/1\text{-}.001 \approx 1.001 = 1 + .001$$

$$1/.998 = 1/1\text{-}.002 \approx 1.002 = 1 + .002$$

$$1/.995 = 1/1\text{-}.005 \approx 1.005 = 1 + .005$$

We can now use this approximation to rewrite our buoyancy correction equation as

$$M_s \approx M_w(1-\rho_a/\rho_w)(1+\rho_a/\rho_s)$$

Now we can carry out the multiplication of the two factors, getting

$$M_s \cong M_w(1+\rho_a/\rho_s - \rho_a/\rho_w - \rho_a^2/\rho_s\rho_w).$$

Finally we make one more approximation by noting that the last term is the product of two small numbers; each of them is of the order 10^{-4}, so that their product is of the order of 10^{-8}. Therefore this last term is negligible compared to the others and may be dropped. Then our buoyancy correction equation becomes

$$M_s \approx M_w\,[1+\rho_a(1/\rho_s - 1/\rho_w)] \qquad (17)$$

(Buoyancy correction equation)

This equation is in a form which is useful for working buoyancy corrections. It shows that the correction factors are close to unity, but the one is already separated out, and the difference term can be easily calculated. Furthermore, the manner in which the densities affect the correction factor can be analyzed.

Question 30. If the density of the sample is the same as the density of the weights, then what is the value of the correction factor?

Question 31. If the density of the sample is less than the density of the weights, is the correction factor less than one or greater than one?

Question 32. If the sample is more dense than the weights, is the correction factor less than one or greater than one?

Example Problem. What is the mass of a sample whose density is one half that of the removable balance weights, if the reading for the sample's mass on an analytical balance is 27.3722 g? The weights are stainless steel with a density of 8.02 g/cm^3.

Solution. Given is:

$$M_w = 27.3722 \text{ g}$$

$$\rho_s = \tfrac{1}{2}\rho_w = 4.01 \text{ g/cm}^3$$

and

$$\rho_a = 1.2 \times 10^{-3} \text{ g/cm}^3$$

Substituting into the buoyancy correction equation,

$$M_s = 27.3722 \text{ g}\left[1 + 1.2 \times 10^{-3} \text{ g/cm}^3 \times \left(\frac{1}{\tfrac{1}{2} \times 8.02 \text{ g/cm}^3} - \frac{1}{8.02 \text{g/cm}^3}\right)\right]$$

Simplifying and cancelling units inside the brackets gives

$$M_s = 27.3722 \text{ g}\left[1 + \frac{1.2 \times 10^{-3}}{8.02}\right]$$

This equation further reduces to

$$M_s = 27.3722 \text{ g} \quad [1 + 0.00015]$$

and our true sample mass is

$$M_s = 27.3763 \text{ g}$$

Problem 15. What is the mass of a sample whose density is one third that of the removable balance weights, if the reading for the sample's mass on an analytical balance is 106.8355 g? The removable weights are stainless steel (ρ = 8.02 g/cm^3).

Example Problem. The mass of a certain sample as read on an analytical balance is 47.1025 g. The sample has a density of 13.0 g/cm^3 which is twice the density of the removable balance weights. What is the true mass of the sample?

Solution. Given are:

$$M_w = 47.1025 \text{ g}, \ \rho_a = 1.2 \times 10^{-3} \text{g/cm}^3$$

$$\rho_s = 13.0 \text{ g/cm}^3 = 2\rho_w$$

Substituting into the buoyancy correction equation gives

$$M_s = 47.1025 \text{ g} \left[1 + 1.2 \times 10^{-3} \text{ g/cm}^3 \times \left(\frac{1}{13.0 \text{ g/cm}^3} - \frac{1}{\frac{1}{2} \times 13.0 \text{ g/cm}^3} \right) \right]$$

Simplifying this expression

$$M_s = 47.1025 \text{ g} \; [\, 1 - (1.2 \times 10^{-3})/13 \,]$$

which reduces to

$$M_s = 47.1025 \text{ g} \; [\, 1 - 0.000092 \,]$$

and finally to

$$M_s = 47.0982 \text{ g}$$

Question 33. When the density of a sample is less than the density of the removable balance weights in a substitution balance, will the corrected mass be larger or smaller than the balance reading? When the density of the sample is greater than the density of the removable weights, will the corrected mass be larger or smaller than the balance reading?

Problem 16. The mass of a sample is read on an analytical balance as 2.1078 g. The sample has the same density as that of the removable weights. What is the corrected mass of the sample?

CLASSIFICATION OF ERRORS

In general, errors in measurements of any kind can be classified into two groups: systematic and random. *Systematic errors* are produced by defects in the measuring instrument or in its use. These errors tend to be such that they are always too high or always too low in every reading. Such systematic errors often may be eliminated by correcting the defect. Examples of sources of systematic error in an analytical balance are corrosion of a weight ring, an inaccuracy in the graduation plate, or the buoyancy of air (if no correction is made for this effect).

Random errors, on the other hand, arise from unpredictable causes and usually will be different on different readings. They are impossible to eliminate completely. Sources of random error in the analytical balance may be static electricity, small air currents inside the instrument, or variations of the lengths of components due to temperature changes. Because random errors are present in every kind of measurement, methods have been devised for minimizing their effect on the final result, and for estimating their magnitude. The latter task is every bit as important as the former. In science it is just as necessary to have an estimate of your error as it is to make the error small.

EXPERIMENT C-2. The Analytical Balance

Write answers to questions, and complete the table on the worksheet at the back of this module.

In this experiment you will make a series of measurements on a substitution-type analytical balance. Such instruments are sensitive and expensive. You should therefore read the instructions on the balance, or get such instructions from your teacher, before starting to use it.

1. A little later in this experiment you will be asked to weigh ten copper pennies, which have a density of 8.89 g/cm^3. You will need to know the density of the removable weights in your balance. For weighing a copper penny, with stainless steel removable weights (8.02 g/cm^3), the corrected mass is then

$$M_s = M_w\,[\,1 + 1.2 \times 10^{-3}(1/8.89 - 1/8.02)\,]$$

or

$$M_s = M_w\,[\,1 + 1.2 \times 10^{-3}\,(-0.87/71.3)\,]$$

or

$$M_s = M_w(1 - 1.5 \times 10^{-5})$$

$$= M_w - 1.5 \times 10^{-5} M_w$$

$$= M_w - 0.000015 M_w$$

You have also been given a small piece of glass (density = 2.6 g/cm^3) and a piece of lead (density = 11.3 g/cm^3).

Calculate the correction for each of these substances in your work sheet.

2. Weigh the piece of glass, and use the correction to find the corrected mass, M_s, for glass.

3. Weigh the piece of lead, and use the correction to find the corrected mass, M_s, for lead.

4. Now that you have had a little experience using the substitution analytical balance, you will make some careful weighings of the copper pennies.

   Weigh each of the pennies in the following way: place the penny on the pan, weigh it carefully, remove the penny, then prepare the balance as though you were going to weigh some other object. Weigh the next penny. In other words, do not let the mass of one penny influence what you read as the mass of another penny.

   Make these weighings and use Table I to show these uncorrected masses. You now have a single mass reading for each of ten pennies. Make the buoyancy correction for each of these masses, and show the corrected masses in Table I.

In Experiment C-2, you weighed ten different pennies on a substitution type analytical balance. When we made a similar set of measurements, we found that each penny had a mass of about 3 g, with a correction term too small to affect the digit in the 4th decimal place. The results of these weighings are:

Table I.

Coin	Mass (g)
1	3.0731
2	3.1097
3	3.0888
4	3.1235
5	3.0753
6	3.1016
7	3.0826
8	3.1158
9	3.0799
10	3.1028

Sum = 30.9531

WHAT IS THE MASS?

What should we say is the mass of a penny? The values are different, so how shall we answer this question? What we are looking for is a single number which best represents the list of values in Table I. It seems reasonable, and it is a universally adopted procedure, to take the average (*arithmetic mean*) as the best representative value. One should be aware, however, that there exists no proof that the average is in general the best value. For certain kinds of errors the mean can be proved to be the best value, statistically, but our habit of taking it as best is accepted more by common practice and by the fact that it seems to make sense, than by a general proof.

Problem 17. Find the average mass of the ten pennies whose corrected masses are shown in Table I. You find the average by finding the sum of the masses and dividing by however many pennies you have.

Are the variations in the masses of pennies in Table I due to the pennies having different masses, or are these variations due to the balance? What results did you get when you weighed the same penny 10 times? When we did that weighing carefully, we got the same result every time, 3.0753 g. If you also got identical results on repeated weighings of the same penny, then variations in mass observed for different pennies must be because the pennies actually have different masses.

THE ARITHMETIC MEAN

Suppose that we must tell someone the mass of a penny. We have already seen that these masses vary, so what answer can we give? The average mass is our best estimate of the mass of a penny. We call this average the arithmetic mean. You already know how to find the mean. You add all the masses and divide by how many you had. In mathematics, there is a symbol which is a shorthand for the instruction "add up the following list of quantities." This symbol is the capital Greek letter *sigma,* written Σ. To indicate that an addition is to be performed, you place a symbol for the quantities to be added following Σ. Thus, Σm means add all the masses.

Usually when a sum is to be found, there are a certain number of readings. In your experiment, you had ten readings. The sum could be stated symbolically by the statement,

$$\Sigma m = m_1 + m_2 + m_3 + m_4 + m_5 +$$
$$m_6 + m_7 + m_8 + m_9 + m_{10}$$

You can see that the left side of the equation is much easier to write than the ten individual terms. However, the left side doesn't tell us how many terms tere are. To indicate that number, we modify the symbol Σ in the following way.

$$\text{Sum} = \sum_{i=1}^{10} m_i$$

You would read this as "the sum of m_i, as i goes from 1 to 10." It means, start with m_1, add it to m_2, and continue until you have included m_{10}.

Problem 18. Suppose you have weighed a small sample 7 times, with the following results: 0.0123 g; .0125 g; .0122 g; .0125 g; .0127 g; .0124 g; .0125 g. Find the following sums:

$$\sum_{i=1}^{3} m_i \qquad \sum_{i=1}^{5} m_i$$

Now that you understand the symbol for summing, let's again define the arithmetic mean using this symbol. We already know that the mean is given by

$$\text{Mean} = \frac{\text{sum of masses}}{\text{number of masses}}$$

But the sum of masses can be written

$$\sum_{i=1}^{N} m_i$$

for N weighings, and the mean is usually represented by the symbol $\overline{m}$. We therefore have

$$\overline{m} = \frac{\sum_{i=1}^{N} m_i}{N}$$

as our definition of the mean value of m.

DEVIATIONS FROM THE MEAN

You've already found the mean for the measurements of Table I. The amount by which each of our measurements differs from the mean is a measure of how "spread out" our measurements are. For example, the mass of coin 2 differs from the mean by 3.1097 g − 3.0953 g = +0.0144 g, and the mass of coin 7 differs by 3.0826 g − 3.0953 g = −.0127 g. Since we are interested only in how far each measurement is from the mean, we shall ignore the sign difference. The result arrived at after you "ignore the sign" is called *absolute value.* For example, −.0127 has the absolute value .0127. Absolute value is shown by placing vertical bars on both sides of a number (|number|). Using absolute values, we define the *deviation from the mean* for each of the preceding mass measurements as

$$d_i = |m_i - \overline{m}|$$

Problem 19. Use the definition of d_i, the deviation from the mean, to calculate the deviation of each mass from the mean for Table I. Calculate the average of these deviations.

Examples of Measurements

Look at the sets of experimental measurements shown in Table II. We have made these up in order to show certain features that arise in the interpretation of numerical results. The measurements might be of any quantity: distance, mass, temperature change, speed, force, electric current. Each case represents a set of measurements (say, of speed) made in one situation with one set of instruments. The problem is to compare the results obtained in the different cases: do they agree or disagree with each other? Are the differences significant? (What do we mean by "significant" for this kind of problem?)

Table II

Case I

Measurement	Deviation from Mean
2.47	.03
2.54	.04
2.53	.03
2.46	.04
2.52	.02
2.48	.02
Sum 15.00	.18

Average:	
2.50	.03

Case II

Measurement	Deviation from Mean
2.57	.00
2.56	.01
2.58	.01
2.55	.02
2.59	.02
2.57	.00
Sum 15.42	.06
Average:	
2.57	.01

Case III

Measurement	Deviation from Mean
2.58	.11
2.42	.05
2.52	.05
2.38	.09
2.40	.07
2.52	.05
	.42
Sum 14.82	
Average:	
2.47	.07

The Scatter or Spread

In reporting the result of a set of measurements, we almost always give the average value. Then we describe the "scatter" or "uncertainty" of the set in any one of several ways which are aimed at giving the reader a notion of the scatter without listing every one of the measured values. Let us illustrate with Case I of Table II:

1. We might report the average value together with the highest and lowest values that were measured. In that case we list and label three numbers:

Highest value	2.54
Average value	2.50
Lowest value	2.46

As indicated earlier, we always take the average value (2.50 in this case) as the best representation of the measurements on the chance that the "true" or "correct" value is close to average, but we recognize that the truth may possibly be as high as 2.54 or as low as 2.46, or even beyond these values.

2. Another way of reporting the result is to add the average deviation to the mean to get the upper value and subtract the average deviation from the mean to get the lower value. This range, from 2.47 to 2.53, can be written

$$2.50 \pm .03$$

3. Still another way of reporting the result is to show the average deviation as a percentage:

$$0.03/2.50 = 0.012$$

Thus 0.03 is 1.2% of 2.50. We then write the result as

$$2.50 \pm 1.2\%$$

Problem 20. Express the results of Cases II and III in Table II in each of the three ways illustrated above.

Now let us examine percentage deviations in the three cases of Table II:

Case	Average Deviation (%)
I	1.2
II	0.4
III	2.8

Case II shows the smallest scatter while Case III shows the largest. We say that the measurements in Case II have the highest *precision* among the three cases, while those of Case III show the lowest precision. High precision is associated with low scatter.

When Are Means Different?

Suppose that Cases I and III of Table II represent measurements of the speed of a car on two separate occasions. Consider the following: the average speed the first time was 2.50 m/s and the second time 2.47 m/s as far as the tables of numbers are concerned. Can we say that the speeds on the two occasions were actually different, with a lower speed the second time?

To answer this question, let us examine the information in the deviations (or uncertainties):

Case I	2.50 ± 0.03
Case III	2.47 ± 0.07

In other words, the truth in Case I might well be as low as 2.47, while that in Case III might be as high as 2.54. The values overlap. We cannot say that the speeds in Cases I and III were different. We describe the only conclusion we can reach in the following way:

> The results in Cases I and III agree with each other within the scatter (or experimental uncertainty).
>
> or
>
> The results in Cases I and III do *not differ significantly* (because of the experimental scatter).

Let us compare Cases I and II in a similar way to determine whether the results agree or differ from each other.

Case I:	2.50 ± 0.03
Case II:	2.57 ± 0.01

The lowest "correct" value we might admit for Case II is 2.56 while the highest for Case I is 2.53. The possibly correct values do not overlap as they did in our previous illustration, and we say:

> The result obtained in Case II is significantly different from (higher than) the result obtained in Case I.
>
> or
>
> The measurements in Cases I and II disagree with each other by a significant amount.

Question 34. Do the measurements in Cases II and III agree or disagree with each other relative to the experimental scatter? Analyze them in the manner illustrated and describe your conclusions in your own words.

PRECISION AND ACCURACY

The results in Table II might be used to illustrate still another situation that may arise in making measurements. Suppose that the three cases represent resistance measurements made on the same resistor with three different ohmmeters. What can we say about the meters? Although meters I and III exhibit a lower precision (greater scatter) than meter II, they agree with each other within experimental uncertainty, while meter II differs.

This might mean (we cannot be absolutely certain in such circumstances) that meter II contains a systematic error, although its random errors are lower than those of the other meters. If this is true, we would say that meter II is *more precise* than meters I and III, but it is *less accurate*.

"Precision" is used to describe *repeatability* of a measurement—the extent to which the measurement is subject to random error. "Accuracy," on the other hand, is used to describe *correctness* of a measurement—the extent to which the measurement is subject to systematic error. It is therefore possible for an inaccurate measurement to be precise.

The various ways in which we have interpreted the data in Table II illustrate some very basic aspects of statistical reasoning. In giving these illustrations, we have used a very simple, but extremely crude, measure of scatter or uncertainty: the average devia-

tion. The science of statistics defines other measures (standard deviation; probable error) that are mathematically more meaningful than average deviation because they can be interpreted in terms of the *probability* that the true value we are trying to measure lies within a certain range or distance from the calculated average.

IS SCIENCE EXACT?

One more comment should be made about the ideas we have developed: scatter, uncertainty, agreement, disagreement, and significant differences between numerical measurements. Students are sometimes disturbed to find that scientific or technical measurements and calculations are always to some extent uncertain or inexact. They have the false impression that science and engineering are exact when dealing with numbers.

It is important to realize no scientific or engineering measurements are ever exact. There is nothing bad, evil, or deficient about having error and uncertainty in one's results; the important thing is to be aware of the uncertainty, to be able to estimate it, and to inform the user of your data exactly what is known about its size.

It is interesting to realize that the most significant aspect of our numerical knowledge in science and engineering is not the fact that it is exact, but the fact that by keeping track of numerical errors and uncertainties, we have information about how wrong we might be. This may sound peculiar to you at first, but it represents a deep understanding of the nature of technical work.

SUMMARY

For small angular deflections of a balance beam, using a simple theory involving torques, the *sensitivity* of a balance is predicted by theory to be given by

$$E = \frac{DL_p}{M_{beam}d}$$

where D is the pointer length, L_p is the pan arm length, M_{beam} is the mass of the rigid beam system, and d is the distance from the fulcrum to the center of mass of the beam system.

The *density* of a substance is defined as the mass associated with each unit volume of the substance. As an equation, the definition is

$$\rho = M/V$$

where M is the mass and V is the volume.

When an object is immersed in a fluid, or floats on a fluid, the upward force is called a *buoyant force.*

The buoyant force is equal to the weight of the fluid which is displaced. This is called *Archimedes' Principle.* The principle can be stated in the form

$$B = \rho_f V_d g$$

where B is the buoyant force, ρ_f is the density of the fluid, and V_d is the volume of fluid displaced.

When weighing substances on an analytical balance, a buoyancy correction may be made due to the presence of air as a fluid. The correction equation is

$$M_s = M_w\ [\ 1 + \rho_a(1/\rho_s - 1/\rho_w)\ ]$$

where M_s is the corrected mass of the sample, M_w is the balance reading, ρ_s is the density of the sample and ρ_w is the density of the removable weights.

The arithmetic mean of a set of measurements is the sum of the values divided by the number of measurements. In equation form this definition is

$$\overline{m} = \frac{\sum_{i=1}^{N} m_i}{N}$$

where $\overline{m}$ is the mean, m_i is the value of the ith measurement, and N is the number of measurements.

The *deviation,* d_i, of the ith measurement *from the mean* is defined by

$$d_i = |m_i - \overline{m}|$$

where we take the absolute value of the difference between the measurement and the mean of the set of measurements.

The *mean deviation* is the average of the deviations and is given by

$$\bar{d} = \frac{\sum_{i=1}^{N} m_i}{N}$$

where $\bar{d}$ is the mean deviation, d_i is the ith deviation, and N is the number of measurements.

The best value and upper and lower limits of a measurement can be described using the mean as the best value and the mean deviation as the upper and lower limit, i.e.,

$$\text{Best value} = \bar{m} \pm \bar{d}$$

The *precision* of a set of measurements is the extent to which the measurements are repeatable. Mean deviation is a measure of precision. When measurements are all identical, the smallest scale division on the instrument is the measure of precision.

The *accuracy* of an instrument is the degree to which the instrument agrees with an instrument calibrated correctly against a standard.

EXPERIMENT A-1
WORK SHEETS

Name ______________________________

Part A

1. A.
   B.
   C.
   D.

2.

	First Penny	Second Penny
Weight on A		
Weight on B		
Weight on C		
Weight on D		

3.

Balance or Scale	Difference in Weight Readings
A	
B	
C	
D	

4. ______________________________

5.

Balance	Capacity
A	
B	
C	
D	

6. ______________________________

7.

Balance	Weight of First Object	Weight of Second Object	Weight Difference
A			
B			
C			
D			

8. ______________________________

Part B

1. ______________________________

2. ______________________________

3.

4.

5.

Part C

1.

2.

Part D

1

2.

3.

4.

Part E

1.

2.

COMPUTATION SHEET

EXPERIMENT B-1
WORK SHEETS

Name ______________________________

Part A

1. Position of fulcrum = ______cm

2. Mass of left pan support = ______g

   Mass of right pan support = ______g

3. Table I

Trial	M_1	L_1	M_2	L_2
1	100 g	40 cm	200 g	
2	100	30	200	
3	100	40	400	
4	200	15	100	

4. ______________________________

5. ______________________________

Part B

1. Mass of third pan support = ______g

   Mass of third weight holder = ______g

2. Table II

Trial	M_1	L_1	M_2	L_2	M_3	L_3
1	200 g	40 cm	150	40	100	
2	200	32	100	40	120	
3	100	45	150	20	150	
4	200	30	100	30	200	

3. Table III

Trial	$M_1 L_1$	$M_2 L_2$	$M_3 L_3$
1			
2			
3			
4			

4. ______________________________

Part C

1. Position of fulcrum = ______cm

2. Table IV

Trial	M_1	L_1	M_2	L_2
1	200 g	30 cm	150 g	
2	100	20	75	
3	400	15	300	
4	250	10	100	

3. ______________________________

____

____

____

4. Weight of meter stick = ______g

5. ______________________________

____

____

____

____

____

____

6. ______________________________

____

____

____

____

____

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____

____

____

____

EXPERIMENT B-2
WORK SHEETS

Name ______________________

Part A

1. Position of fulcrum = cm

2. Mass of Rigid Part of Beam System =

3. Length of Pointer (from fulcrum to pointer tip) =
   Load on each side of fulcrum = 150 grams

Table V

Trial	Additional Mass, m_o in grams	Deflection s produced by m_o (pan arm = 15 cm)	Deflection s produced by m_o (pan arm = 25 cm)	Deflection s produced by m_o (pan arm = 35 cm)	Deflection s produced by m_o (pan arm = 45 cm)
1	2				
2	4				
3	6				
4	8				
5	10				

4.

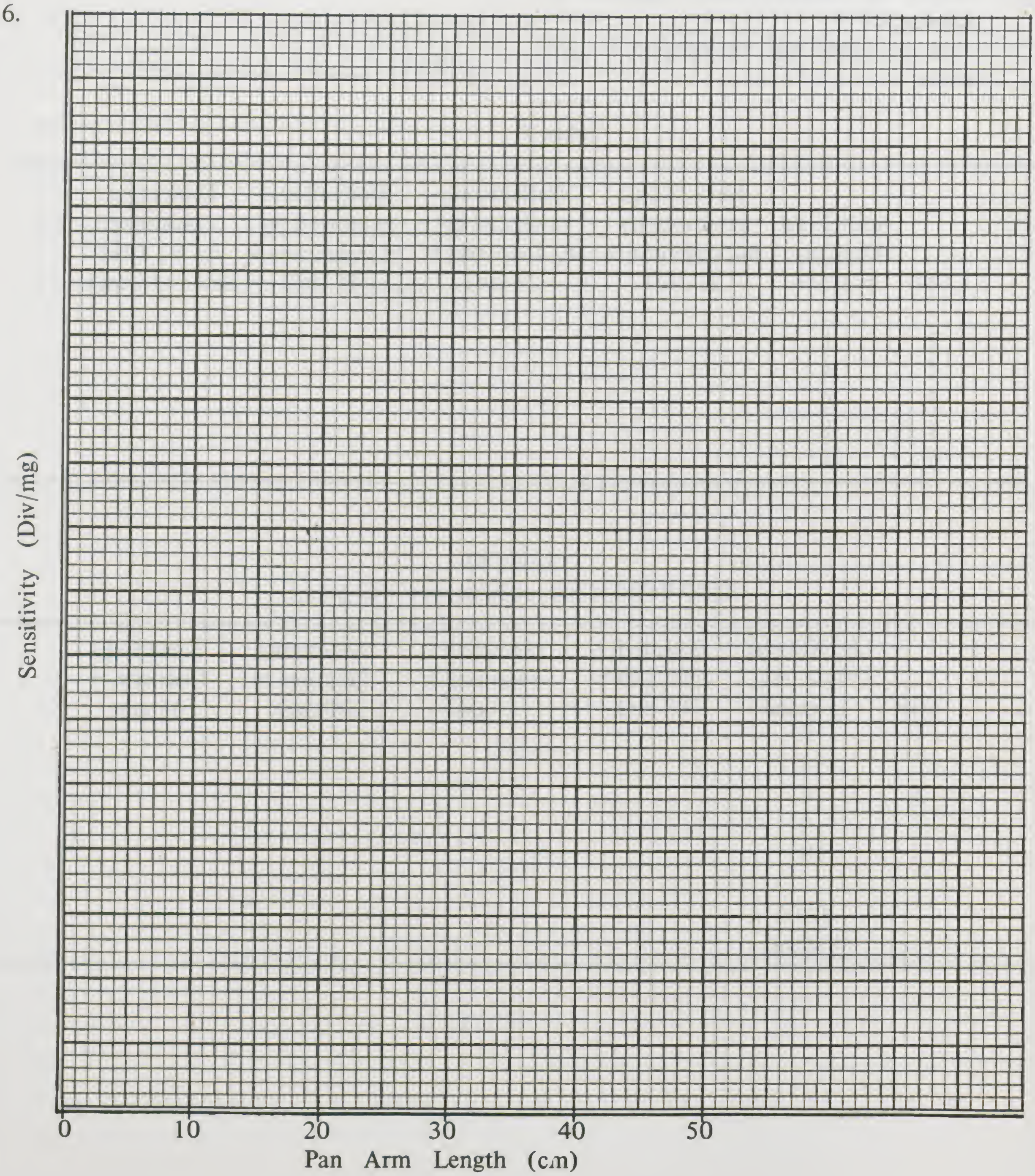
Table VI
Load on each side of fulcrum = 150 grams

Trial	Additional Mass, m_o, in grams	Sensitivity (pan arm = 15 cm)	Sensitivity (pan arm = 25 cm)	Sensitivity (pan arm = 35 cm)	Sensitivity (pan arm = 45 cm)
1	2				
2	4				
3	6				
4	8				
5	10				
	Total				
	Average				

5. Table VII

Average Sensitivity (div/mg)	Pan Arm Length (cm)
	15
	25
	35
	45

6.



7. ______________________________

____

____

____

____

8. ______________________________

Part B

1. $M =$

2. $M_{\text{beam system}} = 2M =$ __________

4. $2d =$ (fulcrum to center of movable weight)

   $d =$ (fulcrum to center of mass of beam system)

5.

Table VIII

Trial	Added Mass m_o	Scale Deflections	Sensitivity (div/g)	Sensitivity (div/mg)
1	g	div	div/g	div/mg
2				
3				
4				
5				
			Total =	

Pan arm length = 45 cm

$d =$ ______

Average Sensitivity = $\frac{\_\_\_\_}{5}$ = ______

6. $d =$ ______
   $E =$ ______

7. $d =$ ___ $d =$ ___
   $E =$ ___ $E =$ ___

8.

E (div/mg)	d (cm)

10. ______________________________

____

____

____

____

____

____

____

____

____

____

____

11.

E (div/mg)	$1/d$ (1/cm)

13. E = ________

COMPUTATION SHEET

EXPERIMENT C-1
WORK SHEETS

Name ____________________

1.

	Al	Zn	Sn	Cu	Pb
L					
d					

2, 3, 4, 6.

Table I

Sample	1 Mass Reading for sample in air M_a	2 Mass Reading for sample in water M_w	3 Calculated Volume of sample (same as volume of water displaced	4 Difference of mass readings $M_a - M_w$	5 Mass of water displaced $\rho_w V_w$
	grams	grams	cm^3	grams	grams
1					
2					
3					
4					
5					

5. $M =$ ________

$\rho = \frac{M}{V} =$ ______ $=$ ______ g/cm^3

7. ____________________________________

____

____

COMPUTATION SHEET

EXPERIMENT C-2
WORK SHEETS

Name ______________________

1. ____

____

____

____

____

2. M_W = ________(uncorrected)

M_s = ________(corrected)

3. M_W = ________(uncorrected)

M_s = ________(corrected)

4.

Table I

Masses of
Ten Different Pennies

Penny Number	Uncorrected Mass (grams)	Corrected Mass (grams)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

COMPUTATION SHEET

07-001711-5